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Local Lorentz Transformations and Paradoxes

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ABSTRACT: In previous work it was shown that instead of the known Lorentz Transformations (LT) the new obtained Vectorial Lorentz Transformations (VLT) were the transformations that truly respected the Principle of Relativity and the consideration of the speed of light as a universal constant. In this work is presented a new view that clarifies how it is possible to obtain practical consequences of the VLT, and its applications to our real life.

KEYWORDS: Special Relativity, Lorentz Transformations, Vectorial Lorentz Transformations, Local Lorentz Transformations, Time Dilation, Length Contraction, Twin, Ladder and Bell paradoxes.

I INTRODUCTION

As it is commonly expressed in the relativistic literature, some of the practical consequences of Lorentz Transformations (LT), under conditions of simultaneity of events, or their occurrence at the same location, are those known as length contraction and time dilation, respectively. In this work such concepts are re-evaluated.

In order to analyze the results or relationships obtained from the Vectorial Lorentz Transformations, or corrected Lorentz Transformations, let's recall the existent conditions within the analysis of such transformations:

a) Two inertial systems with relative movement between them and an observer placed in each system with equipment to measure time and distances, previously calibrated, are considered.

b) Because of the relativity of motion, each observer considers his system as fixed and the other moving at a constant velocity. In this situation and without any other reference, it is impossible to demonstrate which system is moving, as it was shown by Einstein in 1905 [1].

c) When both inertial systems coincide, a light pulse is sent to the space and measurements of its trajectory are done by each observer within his inertial system "without any knowledge of the other observer located in the other inertial system". Thus, each observer measures from his origin of coordinates a different radio-vector to the point P in the space, previously chosen, where the light pulse arrives at.

The following postulates are taking into account in this analysis:

1) Each observer, in an independent way measures that the speed of the light pulse is c.
2) Physical laws should be the same in all inertial reference frames.

We have seen before in the derivation of the Vectorial Lorentz Transformations (VLT), that time is a

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vector depending on space coordinates [2]. Although they were derived by taking a pulse of light as the projectile, their relationships are valid for any kind of projectile moving at a speed distinct of that of light. Such transformations (VLT) are:

$$\mathbf{r}'=\frac{\mathbf{r}-v.\mathbf{t}}{\sqrt{1-\frac{v^2}{c^2}}};\quad \mathbf{t}'=\frac{\mathbf{t}-\frac{v}{c^2}.\mathbf{r}}{\sqrt{1-\frac{v^2}{c^2}}};\quad \mathbf{u}'=\frac{d\mathbf{r}-v.d\mathbf{t}}{\left|d\mathbf{t}-\frac{v}{c^2}.d\mathbf{r}\right|};\quad \mathbf{r}=\frac{\mathbf{r}'+v.\mathbf{t}'}{\sqrt{1-\frac{v^2}{c^2}}};\quad \mathbf{t}=\frac{\mathbf{t}'+\frac{v}{c^2}.\mathbf{r}'}{\sqrt{1-\frac{v^2}{c^2}}}\quad \mathbf{u}=\frac{d\mathbf{r}'+v.d\mathbf{t}'}{\left|d\mathbf{t}'+\frac{v}{c^2}.d\mathbf{r}'\right|} \tag{1}$$

$$\mathbf{r}=\frac{\mathbf{r}'+v.\mathbf{t}'}{\sqrt{1-\frac{v^2}{c^2}}};\quad \mathbf{t}=\frac{\mathbf{t}'+\frac{v}{c^2}.\mathbf{r}'}{\sqrt{1-\frac{v^2}{c^2}}}\quad \mathbf{u}=\frac{d\mathbf{r}'+v.d\mathbf{t}'}{\left|d\mathbf{t}'+\frac{v}{c^2}.d\mathbf{r}'\right|};\quad \mathbf{r}'=\frac{\mathbf{r}-v.\mathbf{t}}{\sqrt{1-\frac{v^2}{c^2}}};\quad \mathbf{t}'=\frac{\mathbf{t}-\frac{v}{c^2}.\mathbf{r}}{\sqrt{1-\frac{v^2}{c^2}}};\quad \mathbf{u}'=\frac{d\mathbf{r}-v.d\mathbf{t}}{\left|d\mathbf{t}-\frac{v}{c^2}.d\mathbf{r}\right|}; \tag{2}$$

Observe: $\mathbf{u}'=\frac{d\mathbf{r}'}{dt'}=\frac{d\mathbf{r}-v.d\mathbf{t}}{\left|d\mathbf{t}-\frac{v}{c^2}.d\mathbf{r}\right|}=\frac{\frac{d\mathbf{r}}{dt}-v.\frac{d\mathbf{t}}{dt}}{\left|\frac{d\mathbf{t}}{dt}-\frac{v}{c^2}.\frac{d\mathbf{r}}{dt}\right|}=\frac{\mathbf{u}-\mathbf{v}}{\left|\frac{d\mathbf{t}}{dt}-\frac{v}{c^2}.\mathbf{u}\right|}$ and $\mathbf{u}=\frac{d\mathbf{r}}{dt}=\frac{d\mathbf{r}'+v.d\mathbf{t}'}{\left|d\mathbf{t}'+\frac{v}{c^2}.d\mathbf{r}'\right|}=\frac{\mathbf{u}'+\mathbf{v}'}{\left|\frac{d\mathbf{t}'}{dt}+\frac{v}{c^2}.\mathbf{u}'\right|}$

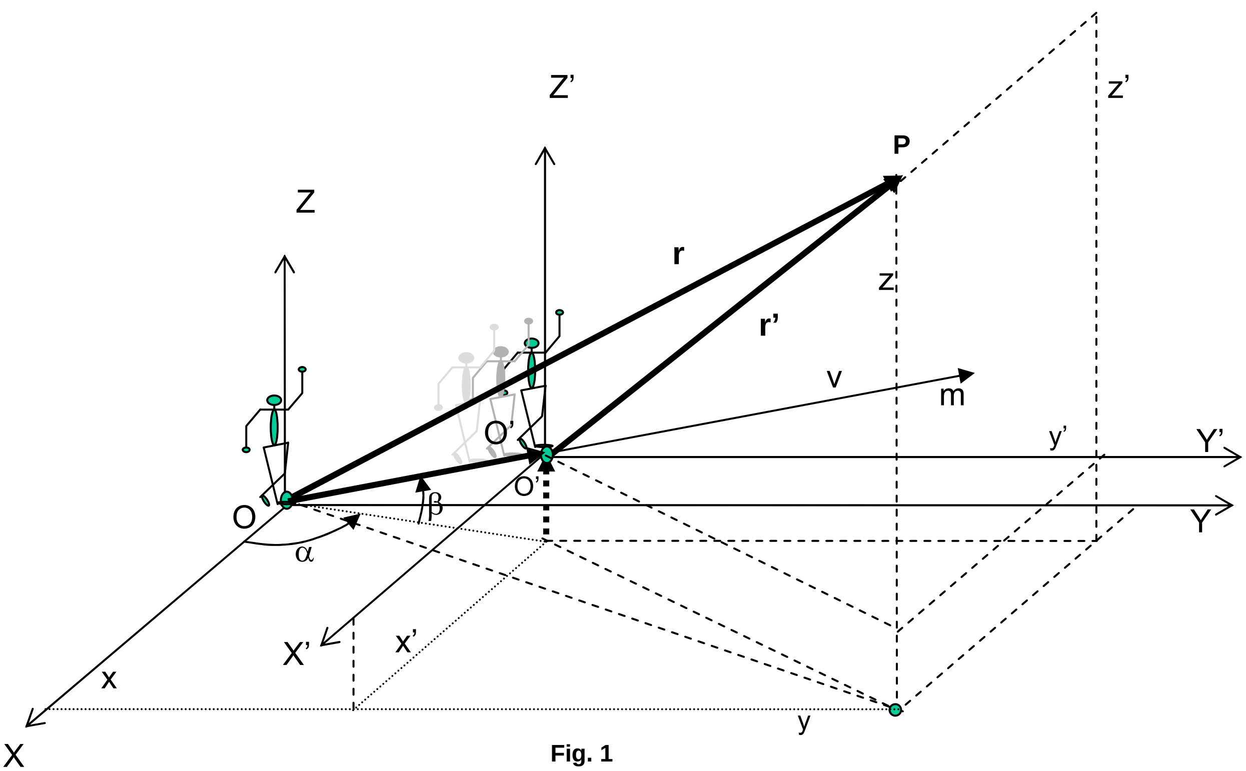

Fig. 1

Expressions of direct (first three) and inverse transformations (last three) in (1) were obtained considering the system O fixed and system O', moving. Similarly, Expressions in (2) are for an opposite configuration, say, now system O' is the system considered fixed and the other one, moving.

In equations (1), speed of moving system is v, but in (2) is $-v$. Observe also that the velocity of the moving system O', relative to the fixed system O in (1), is given by $\mathbf{v}=v.\frac{d\mathbf{t}}{dt}$; and that in equations (2) for a moving system O, and O', fixed, is $\mathbf{v}'=v.\frac{d\mathbf{t}'}{dt}$.

II INTERPRETATION OF VECTORIAL LORENTZ TRANSFORMATIONS

Now, what is the meaning of each one of relationships expressed in equations (1) and (2)?

A. Time Transformations

Obviously, the expressions of the radio vectors $\mathbf{r}$ and $\mathbf{r}'$ are easily obtained from the geometry and symmetry observed in **Fig 1**, but the meaning of $\mathbf{t}$ or $\mathbf{t}'$ are not so clear, because they don't come from a physical or geometrical analysis but from a mathematical derivation, in where we have to search for each variable involved. In this sense, let's try to analyze properly the Vectorial Transformation of time. For starting, let's rewrite the derivation of its transformations (direct and inverse), when point **P** moves at the speed of light, and it is being monitored by two observers with relative motion between them:

$$\mathbf{r}'=\frac{\mathbf{r}-v.\mathbf{t}}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow c.\mathbf{t}'=\frac{c.\mathbf{t}-v.\frac{\mathbf{r}}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \boxed{\mathbf{t}'=\frac{\mathbf{t}-\frac{v}{c^2}.\mathbf{r}}{\sqrt{1-\frac{v^2}{c^2}}}};\ \mathbf{r}=\frac{\mathbf{r}'+v.\mathbf{t}'}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow c.\mathbf{t}=\frac{c.\mathbf{t}'+v.\frac{\mathbf{r}'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \boxed{\mathbf{t}=\frac{\mathbf{t}'+\frac{v}{c^2}.\mathbf{r}'}{\sqrt{1-\frac{v^2}{c^2}}}} \quad (3)$$

$$\mathbf{r}=\frac{\mathbf{r}'+v.\mathbf{t}'}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow c.\mathbf{t}=\frac{c.\mathbf{t}'+v.\frac{\mathbf{r}'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \boxed{\mathbf{t}=\frac{\mathbf{t}'+\frac{v}{c^2}.\mathbf{r}'}{\sqrt{1-\frac{v^2}{c^2}}}};\ \mathbf{r}'=\frac{\mathbf{r}-v.\mathbf{t}}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow c.\mathbf{t}'=\frac{c.\mathbf{t}-v.\frac{\mathbf{r}}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \boxed{\mathbf{t}'=\frac{\mathbf{t}-\frac{v}{c^2}.\mathbf{r}}{\sqrt{1-\frac{v^2}{c^2}}}} \quad (4)$$

It can be realized that the expressions of time $\mathbf{t}$ and $\mathbf{t}'$ in (3) and (4) measured by observers, depends on the measured radio-vector of the projectile.

In order to try to clarify from a physical point of view this aspect of the VLT let's focus, by now, our attention in equation (3) and recall the following example: Let $\mathbf{r}$ be the variable radio-vector of an aircraft in rectilinear inclined flight, measured by an observer at the origin O of a fixed coordinate system, and Let $\mathbf{r}'$ be the radio-vector of the aircraft position, measured by another observer located at the origin O' of a system that moves at speed v relative to system O, see **Fig 1.**

About the expressions of time $\mathbf{t}'$ in (3) we can say that this is the time measured by the observer of the moving system as if he were at the extreme point **P** of the radio-vector $\mathbf{r}$ (which is the same extreme of $\mathbf{r}'$) and not at the origin O', as it is usually understood. Why we assert this?. Well, from expression of time $\mathbf{t}'$ in (3) it is observed that its value depends explicitly on the value of the radio-vector $\mathbf{r}$. This single characteristic suggests its own explanation, because if it were not depending on the radio-vector $\mathbf{r}$, value of $\mathbf{t}'$ would be the same for any value of radio-vector $\mathbf{r}$, and it's not.

On the contrary, for any point **P** _at rest_ relative to O' located ***in any place*** of this moving system, but which travels at constant speed v relative to O, the relationship about time being measured by one and other observer becomes independent of the value of $\mathbf{r}$. In effect, given that **P** moves at speed v, its radio-vector will be given by $\mathbf{r} = v.\mathbf{t}$, then by substituting it in the VLT we have:

$$\mathbf{t}' = \frac{\mathbf{t} - \frac{v}{c^2}.\mathbf{r}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\mathbf{t} - \frac{v}{c^2}.v.\mathbf{t}}{\sqrt{1-\frac{v^2}{c^2}}} \quad \Rightarrow \quad \mathbf{t}' = \sqrt{1-\frac{v^2}{c^2}}.\mathbf{t} \tag{5}$$

This time $\mathbf{t}$ is referred to the point **P**. As a particular case, let the point **P** be located _at rest_ ***at the origin of coordinates of the moving system.*** So, because time is measured at this special point we can then refer to it as a local time at O'.

$$\mathbf{t}'_{O'} = \frac{\mathbf{t} - \frac{v}{c^2}.\mathbf{r}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\mathbf{t} - \frac{v}{c^2}.v.\mathbf{t}}{\sqrt{1-\frac{v^2}{c^2}}} \quad \Rightarrow \quad \mathbf{t}'_{O'} = \sqrt{1-\frac{v^2}{c^2}}.\mathbf{t} \tag{6}$$

$$\Rightarrow \quad \mathbf{t} = \frac{\mathbf{t}'_{O'}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \text{(Time Dilation))} \tag{7}$$

On the other side, for the same point **P** located at rest at the origin of coordinates of the moving system, displacing at a speed v relative to O, but now using the inverse transformation obtained from the VLT in (3), let's try to obtain the same relationship in (7), as a check of consistency:

$$\mathbf{t} = \frac{\mathbf{t}'_{O'} + \frac{v}{c^2}.\mathbf{r}'}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\mathbf{t}'_{O'} + \frac{v}{c^2}.(0)}{\sqrt{1-\frac{v^2}{c^2}}} \quad \Rightarrow \quad \mathbf{t} = \frac{\mathbf{t}'_{O'}}{\sqrt{1-\frac{v^2}{c^2}}} \quad \Rightarrow \quad \mathbf{t}'_{O'} = \sqrt{1-\frac{v^2}{c^2}}.\mathbf{t} \qquad \text{(Time contraction at O')} \tag{8}$$

This result indicates that the analysis has been correctly carried out: A fixed observer at O realizes that his measured time is greater than that given by a clock located at moving origin O' or that he experiments a time dilation relative to that measured by the other observer at O', (7). And conversely, observer located at the origin O', which is conscious he is moving with respect to the fixed origin, appreciates that its measured time experiments the corresponding time contraction relative to O, specified by the result obtained from applying inverse transformation in equation (8).

Let's change the observers role. Point **P** will be now located at rest at the origin of coordinates of a moving system O, displacing at $-v$ relative to, O', fixed, the VLT of time that apply are those in equations (4). We should obtain the opposite result to that (7).

$$\mathbf{t}_O = \frac{\mathbf{t}' + \frac{v}{c^2}.\mathbf{r}'}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\mathbf{t}' + \frac{v}{c^2}.(-v.\mathbf{t}')}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \mathbf{t}_O = \sqrt{1-\frac{v^2}{c^2}}.\mathbf{t}' \qquad (9)$$

$$\Rightarrow \quad \mathbf{t}' = \frac{\mathbf{t}_O}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \text{(Time Dilation at O')} \qquad (10)$$

And applying the Inverse Transformation, it is obtained consistently:

$$\mathbf{t}' = \frac{\mathbf{t}_O - \frac{v}{c^2}.\mathbf{r}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\mathbf{t}_O - \frac{v}{c^2}.(0)}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \mathbf{t}' = \frac{\mathbf{t}_O}{\sqrt{1-\frac{v^2}{c^2}}} \quad \Rightarrow \quad \mathbf{t}_O = \sqrt{1-\frac{v^2}{c^2}}\mathbf{t}' \qquad \text{(Time contraction at O)} \qquad (11)$$

In this way and at this moment of the analysis, we can establish that the *time* $t' = T$ *at the origin of a moving system O',* is measured by a fixed observer at O as a delayed real time, $t = \frac{T}{\sqrt{1-\frac{v^2}{c^2}}}$ as it is indicated by equations (7), (8) and (10), (11) without any doubt. In this sense, a physical experiment can be conducted to corroborate the time dilation fact (and it has been conducted successfully in the 1971-Experiment of Hafele-Keating) predicted also by this theoretical analysis of the VLT, because all the conditions for measuring local times are precisely established and known.

It is important to set up that the last limitation (cursive letters) in the principle stating that **"it is impossible to know if an inertial system moves with respect to another B or if in fact this last one moves with respect to A, *if any other reference is not well-known*"**, deduced by Einstein in [1] (Newton and Galileo, also established it before), is completely correct as it was demonstrated physically and mathematically by the results previously obtained with the application of the direct and inverse transformations of the VLT. To Determine which is the moving system and which is that one to consider fixed is the track to solve any arisen paradox and this is possible in this analysis.

We have presented in a detailed manner this analysis in order avoid some confusions that usually arise referred to the misunderstanding of difference between the inverse Vectorial transformation and the direct one, when they are not referred to the same event, but to a distinct one. Paradoxes have come to the surface, and they have come out a lot, since relativity was successfully launched by Einstein and when relativistic formulas were applied without care to cases where they are not applicable.

From what we have seen, we can talk then about several possible points where a different time can be measured for a same event occurring near two observers moving with relative motion, i.e.: at the fixed origin O with a clock measuring $\mathbf{t}$; another clock at the moving origin O' measuring

$\mathbf{t}'_{O'} = \sqrt{1 - \frac{v^2}{c^2}}.\mathbf{t}$, and also a third clock inside the flying aircraft moving at some speed $\frac{d\mathbf{r'}}{dt} = \mathbf{u'}$, relative to origin O', measuring: $\mathbf{t}'_P = \frac{\mathbf{t} - \frac{v}{c^2}.\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}}$.

And very similar, we can also talk about several possible points with relative motions, where a different time can be measured in this case, i.e.: at the fixed origin O' with a clock measuring $\mathbf{t}'$; another clock at the moving origin O, measuring $\mathbf{t}_O = \sqrt{1 - \frac{v^2}{c^2}}.\mathbf{t}'$, and also a third clock inside the flying aircraft moving at another speed u' relative to O' (which needs to be considered inside the expression of variable $\mathbf{r}'$), measuring: $\mathbf{t}_P = \frac{\mathbf{t}' + \frac{v}{c^2}.\mathbf{r}'}{\sqrt{1 - \frac{v^2}{c^2}}}$. As it can be realized, VLT formulas changes from one case to another one by changing the roles and meanings of variables, i.e.: $\mathbf{r}'$, $\mathbf{t}'$ changes to $\mathbf{r}$, $\mathbf{t}$, but conversely $\mathbf{r}$, $\mathbf{t}$ changes to $\mathbf{r}'$, $\mathbf{t}'$, and v, changes to $-v$. Nevertheless, although we obtain similar expressions of inverse transformations coming from direct ones the meaning of variables do not change. It is important to have in mind what is going on in order avoid confusions and not getting into contradictions (paradoxes).

Another significant aspect to establish in order to simplify the previous analysis is that point **P** _at rest_ could be referred to any generic point at rest relative to its origin of coordinates in the moving system. This means, as it should be obvious, that if an observer is at rest at the origin O' of a moving system and another two more observers at points $\mathbf{P_1}$ and $\mathbf{P_2}$, being also at rest respect to O', all of them will read in their wrist watches the same time, no matter if the system is moving (as a whole at speed v) relative to other system considered fixed. It is important to emphasize this physical aspect of relativistic events. It is as if any point of the moving system behaves as if it were located at the origin O' of the system. So, the structure of the Lorentz factor that relates both measures of time at generic points preserves the same as those between origins O and O'.

By referring to the expressions of time $\mathbf{t}'_{O'}$ in (6) and/or time $\mathbf{t}_O$ in (9), it is noteworthy to say that they constitute a very relevant result, may be the most important one in the analysis of looking for practical applications of VLT, like it is going to be shown as we advance in this development.

B Length Transforamtions

On the other side, let a rod be at rest relative to the origin O' of a moving inertial system located on the X axis, and the measure of the length of such rod is taken by a group of persons who knows that the distance from the extremes A and B of the rod to the origin O', x'_B and x'_A do not change at any time due to it is at rest relative to O'. Because of this, for observer at O' rod's length does not change as time evolves, preserving constant and equal to $x'_B - x'_A = L_0$. This implies that time, although evolving, should be considered the same inside equations of VLT for all points located at rest relative to the origin O', i.e.: $t_B = t_A = t_{O'} = t$:

$$x'_B - x'_A = \frac{x_B - v.t}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{x_A - v.t}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{(x_B - x_A) + v.(t-t)}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{(x_B - x_A)}{\sqrt{1-\frac{v^2}{c^2}}} = L_0 = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (12)$$

$$\Rightarrow \quad L = L_0.\sqrt{1-\frac{v^2}{c^2}} \qquad \text{(length contraction)}$$

In the previous analysis, the consideration of $t_B = t_A = t_{O'} = t$, actually defines the simultaneity inside a system where everything is at rest, and the consideration of the origin O', for previous analysis of time, as the special point to make time measurements, actually defined the same reference .

Previous analysis is not only valid for a rod located on the X-axis, but also for any fixed orientation of the rod at rest inside the system with origin at O'. This system moves as a whole at speed v relative fixed system and considerations of equality of time are also done here. So, the length of the rod preserves constant and equal to L_0, as in fact it is for the observer at O'.

$$\left\|\mathbf{r}'_B - \mathbf{r}'_A\right\| = \left\|\frac{\mathbf{r}_B - v.\mathbf{t}}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{\mathbf{r}_A - v.\mathbf{t}}{\sqrt{1-\frac{v^2}{c^2}}}\right\| = \left\|\frac{\mathbf{r}_B - \mathbf{r}_A}{\sqrt{1-\frac{v^2}{c^2}}}\right\| = \frac{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}{\sqrt{1-\frac{v^2}{c^2}}} = L_0 = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow \quad L = L_0.\sqrt{1-\frac{v^2}{c^2}} \qquad \text{(length contraction)} \qquad (13)$$

Previous result indicates that the length measurements done by the observer located at the origin O of the fixed system give a value that is less than that measured in the moving system, and that rod's contraction measured by the fixed observer is given by a factor which is the same, independently as much of the rod's orientation like of the direction of the system's movement.

Also, as before the value calculated through the inverse transformation, by using $\mathbf{r}' = k.(\mathbf{r} - v.\mathbf{t}) \;\Rightarrow\; \mathbf{r} = \sqrt{1-\frac{v^2}{c^2}}\mathbf{r}' + v.t$ we obtain:

$$\left\|\mathbf{r}_B - \mathbf{r}_A\right\| = \left\|\sqrt{1-\frac{v^2}{c^2}}(\mathbf{r}'_B - \mathbf{r}'_A)\right\| = \sqrt{1-\frac{v^2}{c^2}}\sqrt{(x'_B - x'_A)^2 + (y'_B - y'_A)^2 + (z'_B - z'_A)^2} = L = \sqrt{1-\frac{v^2}{c^2}}L_0$$

$$\Rightarrow \quad L = L_0.\sqrt{1-\frac{v^2}{c^2}} \qquad \text{(length contraction)} \qquad (14)$$

At this moment, it is important to say that what was observed before for time magnitude is similarly noticed for length: a fixed observer at O realizes that his measured rod length is less than that measured at origin O' or, he detects a length contraction relative to that measured by the moving observer at O', according to equation (13). And conversely, observer located at the origin O', who is conscious and in knowledge that he is moving with respect to the fixed origin O, appreciates that its measured rod length experiments a length extension relative to what is measured at O, given by applying the inverse transformation obtained in equation (14). These observations avoids any misunderstanding or falling in contradictions and/or getting into paradoxes in this relativistic analysis.

Suppose now that instead of a bar, a rectangular solid, or a box of sides f, g and h, with a volume $V'_{O'} = V_0 = f.g.h$ located *at rest* relative to the origin O' of the moving system, is measured by the observer at O'. What volume V would be the observer at the origin O of the fixed system measuring? Let f, g and h be expressed as $f = |\mathbf{r}'_B - \mathbf{r}'_A|$, $g = |\mathbf{r}'_C - \mathbf{r}'_B|$, and $h = |\mathbf{r}'_D - \mathbf{r}'_C|$, the distances between points, respectively and each distance be denoted as $|\mathbf{r}_M - \mathbf{r}_N| = \sqrt{(x_M - x_N)^2 + (y_M - y_N)^2 + (z_M - z_N)^2}$. Operating as before, the expression of the volume $V = \|\mathbf{r}_B - \mathbf{r}_A\|.\|\mathbf{r}_C - \mathbf{r}_B\|.\|\mathbf{r}_D - \mathbf{r}_C\|$, measured by the observer at the fixed origin O in function of V_0 would be encountered through:

$$V_0 = \|\mathbf{r}'_B - \mathbf{r}'_A\|.\|\mathbf{r}'_C - \mathbf{r}'_B\|.\|\mathbf{r}'_D - \mathbf{r}'_C\| = \left\|\frac{\mathbf{r}_B - \mathbf{r}_A}{\sqrt{1-\frac{v^2}{c^2}}}\right\|.\left\|\frac{\mathbf{r}_B - \mathbf{r}_A}{\sqrt{1-\frac{v^2}{c^2}}}\right\|.\left\|\frac{\mathbf{r}_B - \mathbf{r}_A}{\sqrt{1-\frac{v^2}{c^2}}}\right\| = \frac{\|\mathbf{r}_B - \mathbf{r}_A\|.\|\mathbf{r}_C - \mathbf{r}_B\|.\|\mathbf{r}_D - \mathbf{r}_C\|}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} = \frac{V}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}}$$

$$\Rightarrow \quad V_0 = \frac{V}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \quad \Leftrightarrow \quad V = V_0.\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}} \quad \text{(volume contraction)} \qquad (15)$$

In case of an area it is similarly obtained:

$$A_0 = \|\mathbf{r}'_B - \mathbf{r}'_A\|.\|\mathbf{r}'_C - \mathbf{r}'_B\| = \left\|\frac{\mathbf{r}_B - \mathbf{r}_A}{\sqrt{1-\frac{v^2}{c^2}}}\right\|.\left\|\frac{\mathbf{r}_B - \mathbf{r}_A}{\sqrt{1-\frac{v^2}{c^2}}}\right\| = \frac{\|\mathbf{r}_B - \mathbf{r}_A\|.\|\mathbf{r}_C - \mathbf{r}_B\|.}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} = \frac{A}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}}$$

$$\Rightarrow \quad A_0 = \frac{A}{\left(1-\frac{v^2}{c^2}\right)} \quad \Leftrightarrow \quad A = A_0.\left(1-\frac{v^2}{c^2}\right) \quad \text{(areacontractin)} \qquad (16)$$

The relevant characteristic of these calculations with a VLT particularized to the case of two inertial observers located at the origin of each one of their systems, which move with relative motion between them, is that their results show how each magnitude has its own characteristic Lorentz factor which doesn't depend on the orientation of the line of relative motion. We have called this particular application of the VLT **Local Lorentz Transformations (LLT)** [4]. On the contrary, the erroneous Lorentz Transformations, as we know, originates different transversal and longitudinal transformations, making complex to analyze differences between measurements of some magnitude by two inertial observers with relative motion between them.

III RELATIVISTIC PARADOXES

1) ***Twin Paradox*** (simplified). In order to check the real result of their discussion, one of the twin brothers launches in a spaceship at a constant speed of one ninth of the speed of light and after he arrives back at the same speed at earth his clock registered a period of three years traveling. The problem is: if for the brother in the spaceship his clock measured a period of three years in the trip, how long would be the measured one by the brother been left on Earth?. The paradox of relativists started with the following statement: Because the relative movement between the systems is at constant speed, they are inertial ones, so, the analysis depends on which system is to be considered fixed: The brother on earth sees that the spaceship moves away at 0.9c relative to him. But, brother in the spaceship sees equally that his brother is going away at the same speed of 0.9c. So, at the end they have two solutions and can not establish which the correct answer is.

According to our development, there is no any paradox at all because second brother started a motion relative to earth. Let's suppose their clocks were previously calibrated at earth and the brother in the spaceship obviously knows that he started a trip in relative motion in relation with Earth to arrive back after he measures three years. So, brother's clock at earth, fixed observer, will obtain in function of what measures his brother in the spaceship, moving observer, by applying equation (7), the following measure:

$$t = \frac{t'_{O'}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{3 \text{ years}}{\sqrt{1-\frac{(0.9c)^2}{c^2}}} = \frac{3 \text{ years}}{\sqrt{1-\frac{(0.9c)^2}{c^2}}} = 6.88 \quad \text{years}$$

Thus, traveling twin is now 3.88 years younger than his sedentary brother.

2) ***The paradox of the long ladder and the short garage.*** Let an person A with 10-meters-at-rest ladder and person B into a Garage with a length of also 10 meters long. So, the stationary ladder fits perfectly in the Garage as it can be checked by person B. The paradox is: if the long ladder is carried lengthwise by the person A, at a constant velocity of 0.9c and he passes through the garage, then, due to the relativity effects, the passing ladder will be shorter than the garage (it easily fits in the garage), as observed by person B, but, person A really sees the Garage coming to him at a constant speed of 0.9c and he would detect that the garage is actually shorter than his ladder (ladder does not fit into the garage).

Same as before, ladder and garage's length are measured at rest previously by persons A and B in the garage and they confirm the equality of both lengths. Later, person A starts a motion at constant speed of 0,9c, moving system, relative to the garage, fixed system. So, person A are always viewing that ladder as having the same 10-meters length, because ladder is at rest relative to him. But the fixed observer, person B in the garage, detects that ladder has contracted. By applying direct transformation in equation (13) the observed length contraction is:

$$L = L_0 \cdot \sqrt{1 - \frac{v^2}{c^2}} = 10.\sqrt{1-(0{,}9)^2} = 4.36 \quad \text{meters}$$

Namely, the ladder, as it is observed by person B, is viewed equally contracted by relativistic effects of relative motion from 10 meters to 4.36 meters, because he is undoubtedly moving relative to the garage. There is no paradox. Remember that the inverse transformation is the same as the direct one because time is the same at any point in the moving system measured by observer located at origin O' and it gives as result the same calculated value.

3) ***Bell's Spaceship Paradox***. (Taken from Wikipedia) In Bell's version of this paradox, two spaceships, which are initially at rest in some common inertial reference frame, are connected by a taut string. At time zero in the common inertial frame, both spaceships start to accelerate, with a constant proper acceleration g as measured by an on-board accelerometer. Question: does the string break - i.e. does the distance between the two spaceships increase? (Answer given by J. S. Bell was: yes)

Let another observer located at the origin O of the inertial reference frame, system to be considered fixed, viewing what is happening with the two spaceships. The spaceships start their motion relative to O.

The condition that the two spaceships are accelerated at the same g simplifies the relativistic analysis of the problem and its solution (g can be variable: Earth moves around the sun with variable acceleration and what is at rest remains at rest, defining Earth as an inertial system), because the spaceships and the string can be seen as a whole as a moving system with only one observer at O', in where everything is at rest relative to this origin O': the spaceships are at rest and also the delicate and taut connecting string. Establishing this as so, all distances are preserved inside this moving system, spaceships are at rest and therefore string does not break, because everything remains at rest. This would be confirmed by the observer at O' who is also at rest. Only the observer located at the origin O of the fixed system sees a shortening of the distance between the extreme front of the first spaceship and the extreme back of the second spaceship and in general of any length at rest within the system, due to the relativistic contraction produced by the system's motion on its internal lengths at rest, i.e: Observer at O sees that all internal distances of moving system diminish proportionally.

According to this work, the fixed observer sees also that taut string doesn't break, the same situation observed by observer at O'. So, Bell's statement that string breaks, is wrong.

III LOCAL LORENTZ TRANSFORMATIONS

We have observed that how in the Lorentz Transformations (LT) analysis as in the VLT approach, to obtain a “clean” length contraction it is necessary to have simultaneity of events. Similarly, to observe a “clean” time dilation it needs the events occur at the same place. But, when simultaneity or same location is not possible to achieve complex expressions for length contraction and time dilation arise (and incorrectly in LT). Obviously, they come from the fact that we are comparing measurements done from two distinct inertial systems which have distinct origins.

What was indicated in previous paragraph was for explaining the foundations for obtaining local measurements of time and length, in order to correctly develop what we have done before in an informal way with the previously defined Local Lorentz Transformations (LLT) as a particular case of the Vectorial Lorentz Transformations (VLT).

The way to achieve this goal was by making both observers to do measurements from same reference, so we are changing the VLT conditions. Undoubtedly, we would be modifying the original conditions under VLT were developed. Remember that the main objective in VLT development was to demonstrate how two inertial observers with relative motion between them measure the same speed of a pulse of light, each one doing such measurements relative to his own reference system. However, it is well known that equalization of the most of conditions is the usual way to compare things. In the LLT case, we only wanted to compare the measurements of a magnitude done by a fixed observer with those done by another moving one, and what is the amount of their difference, if any.

Thus, we will see next that situation of simultaneity of events and occurrence at the same place, or the observed contraction of lengths and time dilation in relativity ***can be obtained by*** modifying the configuration of two distinct inertial observers measuring a physical magnitude respect to distinct references, to the situation of ***measuring the same physical magnitude from the same reference.***

What does this mean? As we know, Vectorial Lorentz Transformations are the relationships between measurements of length and time, done by two different moving observers inside their own frame of reference, without knowing each other. When each observer measures the speed of light they do their measurements taking as reference their own origin of coordinates, the origin O for the fixed observer, and the origin O’ for the moving one with the result is that the value of speed of light obtained by each observer is the same. Next, let’s pose the situation of measurements of physical magnitudes for both observers with the different configuration which we have called Local Lorentz Transformations (LLT).

Let’s establish that each observer knows about the presence of the other observer, and they agree to measure magnitudes during the “same period of time”, by taking as reference the same origin of coordinates, in order to compare their results. These measurements, of course will yield different results if compared with those obtained through Vectorial Lorentz Transformations, which are done by taking different origins of coordinates. We will see also that this new configuration will give us practical relationships that are not dependent on the orientation of the body’s movement.

In order to systematize the ideas and to apply them to a true comparison of measurements of physical magnitudes in order to formally obtain the Local Lorentz Transformations, the two following conventions will be established:

1) Hereafter, both observers will do their measurements by taking the same point of reference. Let the origin of the fixed observer be this reference. For example, if fixed observer O measures the radio-vector $\mathbf{r}$ of a projectile sent to the space, see **Fig. 2**, the observer on the moving system O' will measure a similar radio-vector $\mathbf{R}'$ from this same reference of the fixed observer, such that $\mathbf{R}'$ will fulfill $\mathbf{r}' = \mathbf{R}' - \mathbf{R}'_0$, with the definitions of the variables $\mathbf{R}'$ and $\mathbf{R}'_0$ given below:

$$\mathbf{r}' = \frac{\mathbf{r} - v.\mathbf{t}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\mathbf{r}}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{v.\mathbf{t}}{\sqrt{1-\frac{v^2}{c^2}}} = \mathbf{R}' - \mathbf{R}'_0 \; ; \text{ for } \mathbf{R}'_0 = \frac{v.\mathbf{t}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\mathbf{r}_0}{\sqrt{1-\frac{v^2}{c^2}}} \text{ and } \mathbf{R}' = \frac{\mathbf{r}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{17}$$

Then, the measurement of moving observer is related to that of the fixed one, only by a Lorentz scaling factor. Each physical magnitude has its own characteristic factor.

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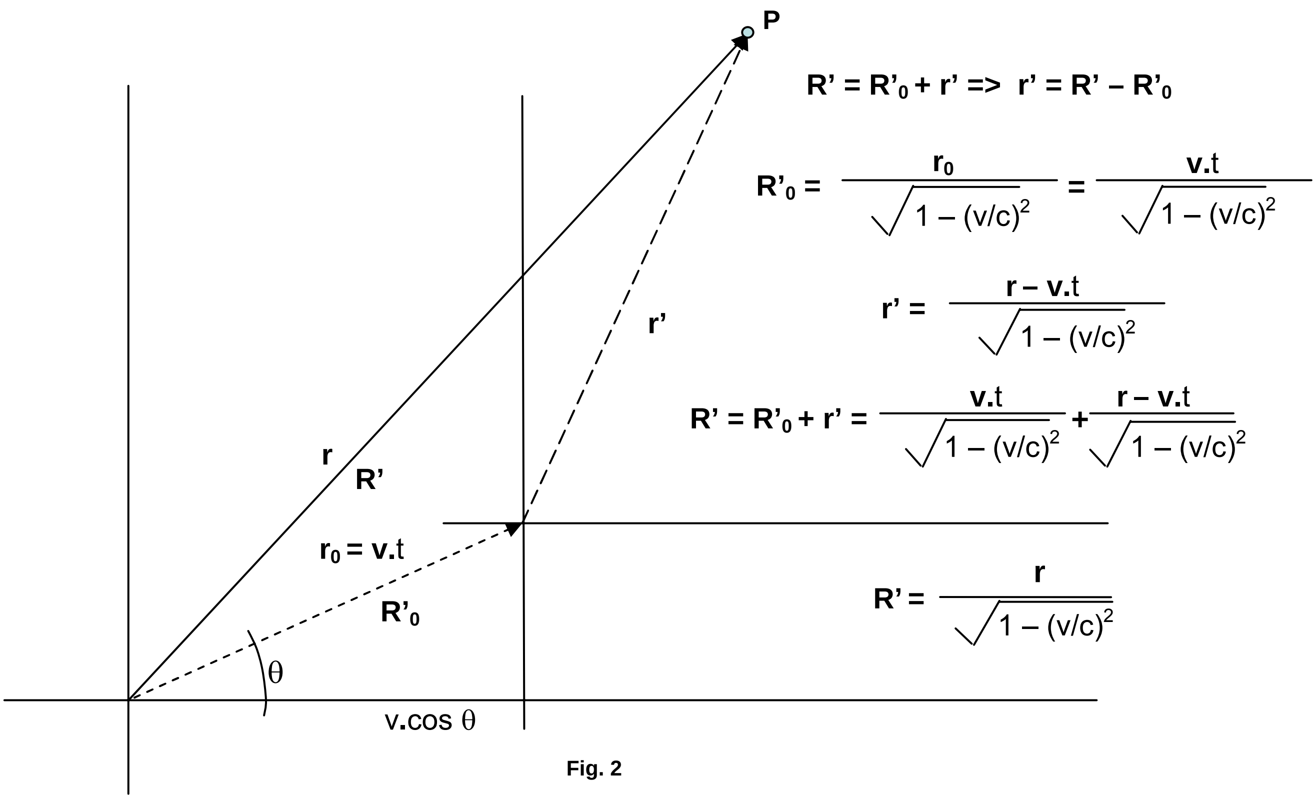

Fig. 2

2) The moving observer O' will not send any pulse of light (or projectile) to any point P, at any instant. So, he will measure a null displacement of projectile, $\mathbf{r}' = 0 \Rightarrow \mathbf{r} = \mathbf{r}_0$ and $\mathbf{R} = \mathbf{R}_0$. Thus, the radio-vector of his moving system, $\mathbf{r} = \mathbf{r}_0$, will be the only measurement completed. Let $\mathbf{t} = \mathbf{T}$ and $\mathbf{t}' = \mathbf{T}'$ the time measured at the origin of each system. Therefore, from the general VLT expressions previously obtained and the conventions applied, the Local Lorentz Transformations (LLT) are obtained as:

$$\mathbf{r}'=0 \Rightarrow \mathbf{r}=v.\mathbf{T}=\mathbf{r_0}; \quad \mathbf{T}'=\frac{\mathbf{T}-\frac{v}{c^2}.v.\mathbf{T}}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \boxed{\mathbf{T}'=\mathbf{T}.\sqrt{1-\frac{v^2}{c^2}}; \quad \mathbf{R}'=\frac{\mathbf{r}}{\sqrt{1-\frac{v^2}{c^2}}}=\frac{v.\mathbf{T}}{\sqrt{1-\frac{v^2}{c^2}}}=\frac{\mathbf{R}}{\sqrt{1-\frac{v^2}{c^2}}}} \quad (18)$$

In sum, measurements of $\mathbf{R'}$, $\mathbf{T'}$, $\mathbf{R}$, $\mathbf{T}$, and $\mathbf{r}=v.\mathbf{T}=\mathbf{R}$, of the origin O' movement now are done by the moving and fixed observers taken as the unique point of reference the origin of coordinates of the system O. As we observe in (18), time and distance vectors are related by a characteristic-scaling factor. The scaling factor, with a value less than unity in the case of time, is a multiplier within each component. For distances, it is a divider. In other words, if any component of a parameter has the same type of factor the contraction or expansion is the same in any direction, for instance, a sphere should expands uniformly in all directions according to the scaling factor depending on its velocity and the speed of light.

However, it is important to point out that the LLT are referred to measurements **relative to the same point of reference: origin O**. This is completely different to what was done for VLT, where each observer did measurements relative to his own reference system. Namely, **the transformations referred as LLT are different to those of VLT.**

With those two conventions in mind, we will not be worried about location coincidence or simultaneity of events. The relations (18) imply that in LLT each physical magnitude observed by an fixed observer, by virtue of its dependency on velocity, in a true way either contract, expand, growth or reduce, with the same scaling factor in all dimensions, independently of their image or shape. For example, if in the moving system an observer at O' measures a bar of length L_0, or if he measures a time t_0 in his measuring, the observer on fixed system at O will measure this length as L and a time as t. The position of the bar in system O' is not relevant; what is important is that it is at rest for the observer at O' and moving relative to O. Thus, the relationship between both measurements, according to LLT, will be:

$$L_0=\frac{L}{\sqrt{1-\frac{v^2}{c^2}}}; \quad t_0=t.\sqrt{1-\frac{v^2}{c^2}} \quad \Rightarrow \quad L=L_0.\sqrt{1-\frac{v^2}{c^2}}; \quad t=\frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad (19)$$

This indicates that an observer in a "stationary" system O measures onto a moving bar at velocity v, a contraction from its original length L_0 to L, no matter which is the position of the bar in the system O', and a time dilation from t_0 to t, as it is shown in equations (19).

Another relevant characteristic is the following one: Lorentz factors in LLT act as scaling factors between measurements done at O and at O', for any magnitude, no matter if this is a differential magnitude or an integral one. In other words, Lorentz factors are simply scaling factors between such measurements.

What is the real meaning of LLT expressed in equations (19)? First of all, **each component is affected by its characteristic Lorentz factor in the same way**, namely, contracting lengths or delaying times. For example, a squared plate with an area at rest $S_0 = L_0^2$, according to equation (16), we will obtain:

$$S' = S_0 = L_0^2 = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}} \cdot \frac{L}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{L^2}{1-\frac{v^2}{c^2}} \Rightarrow S_0 = \frac{S}{1-\frac{v^2}{c^2}} \Rightarrow S = S_0 \cdot \left(1-\frac{v^2}{c^2}\right) \tag{20}$$

And a volume $V' = L_{01}.L_{02}.L_{03}$, measured from O' at rest is related to the volume measured from O, $V = L_1.L_2.L_3$, by the characteristic product of the three contracted lengths is given below in (21):

$$V' = V_0 = L_{01}.L_{02}.L_{03} = \frac{L_1}{\sqrt{1-\frac{v^2}{c^2}}} \cdot \frac{L_2}{\sqrt{1-\frac{v^2}{c^2}}} \cdot \frac{L_3}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{L_1 L_2 L_3}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \Rightarrow V_0 = \frac{V}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}}$$
$$\Rightarrow V = V_0 \cdot \left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}} \tag{21}$$

The vector velocity of the origin O', obtained by differentiating the displacement of O' respect to time will originate, the following LLT:

$$\mathbf{v}' = \frac{d\mathbf{R}'}{dt'} = \frac{\frac{d\mathbf{r}}{\sqrt{1-\frac{v^2}{c^2}}}}{dt.\sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{d\mathbf{r}}{dt}}{1-\frac{v^2}{c^2}} \Rightarrow \mathbf{v}' = \frac{\mathbf{v}}{1-\frac{v^2}{c^2}} \tag{22}$$

At this moment we realize that velocity of the moving system O', plays two roles: either as a scalar, v, when it is inside the scaling factor, in where both observers see each other moving relative to themselves in the same line of relative motion. Or, as a vector $\mathbf{v}'$, measured by the observer at O' into his own frame by taking as reference the origin of the other system O, under the conventions of LLT.

After doing this necessary parenthesis, LLT for acceleration is easily obtained in the same manner as in (22):

$$\mathbf{a}' = \frac{d\mathbf{v}'}{dt'} = \frac{\frac{d\mathbf{v}}{1-\frac{v^2}{c^2}}}{dt.\sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{d\mathbf{v}}{dt}}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \Rightarrow \mathbf{a}' = \frac{\mathbf{a}}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \tag{23}$$

It is also necessary to say at this moment that origin O' could also have a motion along a curvilinear path following a **movement with variable velocity and nevertheless being an inertial system**!. For example Earth has an undoubted inertial curvilinear movement around the Sun, and although it accelerates going to perihelion and reduces its speed after perihelion going to aphelion, we don't feel anything, buildings maintain their verticality, equilibrium of any kind is preserved, etc. So, with the found transformations in equations (22) and (23), we would expect to obtain also LLT in Dynamics. Let's demonstrate next, that it is possible to apply the Vectorial Lorentz Transformations (VLT), to an inertial system of coordinates with curvilinear movement, with respect to a fixed system, located at a point O of the curvilinear trajectory of the moving system.

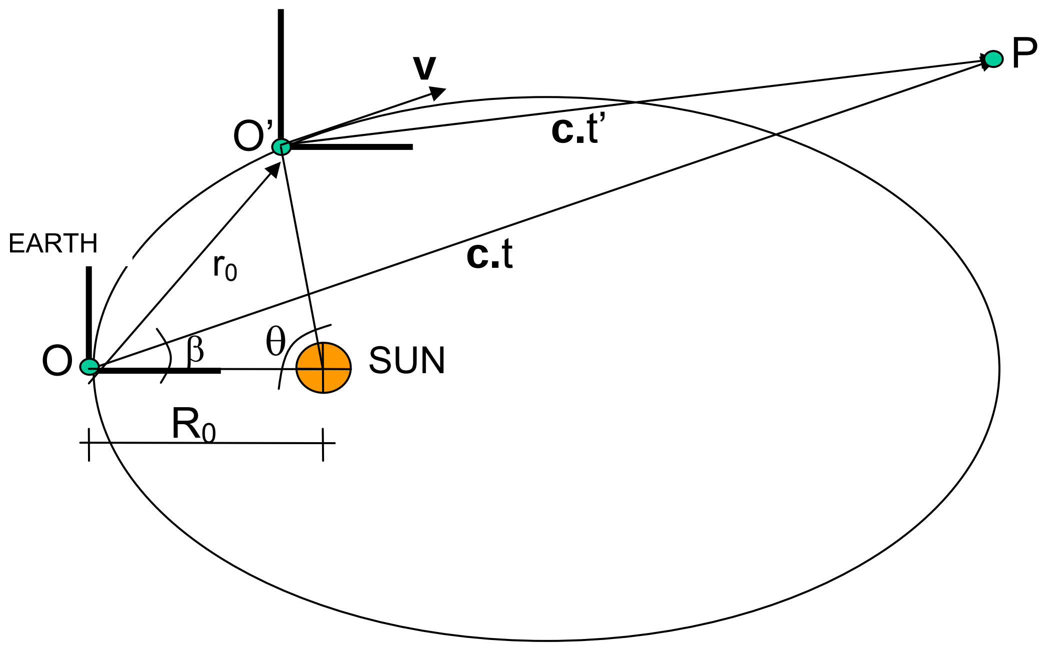

Fig. 3

We can establish that inertial systems are not only those with null acceleration, but those where "the sum of acting forces is null". These include not only those with null acceleration in rectilinear movement, but also those in curvilinear movement with a constant Angular Momentum, where generally acceleration is not constant.

For the movement of Earth around the Sun, the summation of the gravitational force of Sun onto Earth plus the Earth's centrifugal force gives a null result, reason why the Earth movement is inertial according to since it has constant angular momentum. In this way, earth's movement is neither impeded nor eased by any additional external force. We will try to reproduce this movement in **Fig. 3**, where the first observer is on the moving system, Earth, at O', and the second observer will be fixed on the elliptic path at the nearest point to the Sun, the perihelion.

Let R_0 denote the distance between Sun And Earth at the moment when observers start measuring the movement, and r, the generic position of Earth. By taking a closer view at the very beginning of measurements onto this movement, for two dimensions, see **Fig. 4**. Say, when O' and O coincide, a pulse of light is sent forming an angle β with X axis, see **Fig. 3**, and an angle γ between the tangential velocity v of O' with Y axis as it is shown in **Fig. 4**. Let's equally define θ**,** as the angle swept by radius r from $r = R_0$, to the new position r of the moving observer after a period of time dt. At this moment light pulse has reached point P.

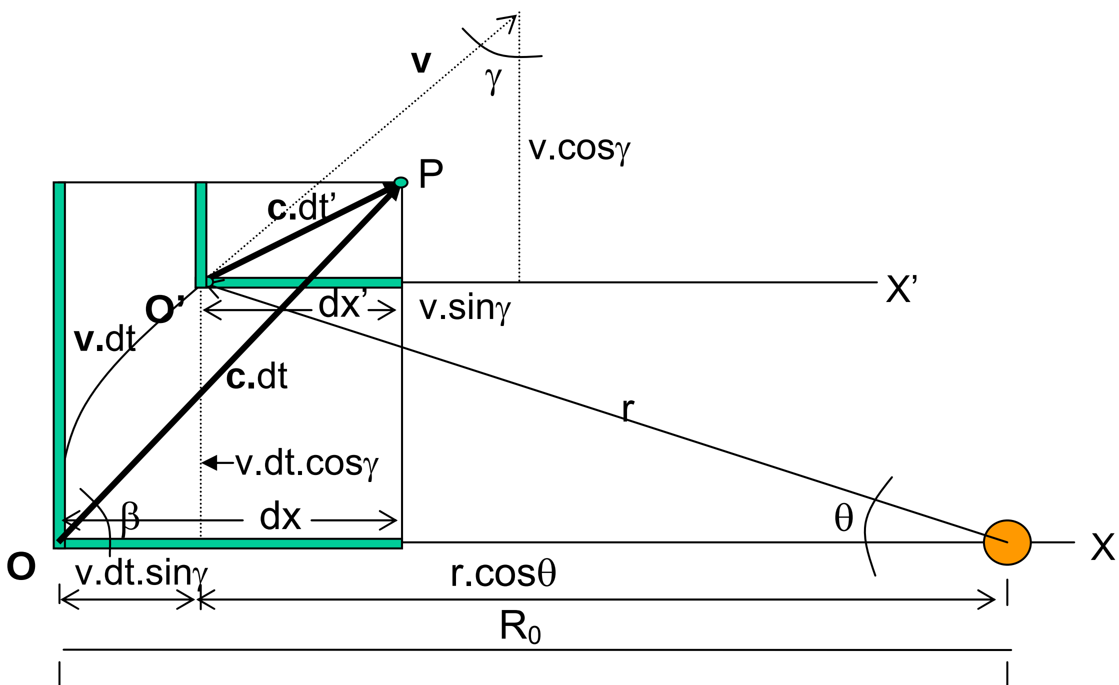

Fig. 4

From **Fig. 3** and **4**, we can establish the following relationships:

$$dx' = k(dx - v.dt.\sin\gamma) \qquad dy' = k(dy - v.dt.\cos\gamma) \tag{24}$$

$$v.dt.\sin\gamma = d(R_0 - r.\cos\theta) \qquad v.dt.\cos\gamma = d(r.\sin\theta) \tag{25}$$

Given that the light speed is the same measured by any observer, it must fulfill:

$$dx^2 + dy^2 = c^2.dt^2 \qquad dx'^2 + dy'^2 = c^2.dt'^2 \tag{26}$$

Substituting dx', dy', by their expressions (24) and (25) into (26), similar expressions previously obtained for rectilinear movement are achieved:

$$dx' = \frac{dx - v.dt.\sin\gamma}{\sqrt{1-\frac{v^2}{c^2}}} \qquad dy' = \frac{dy - v.dt.\cos\gamma}{\sqrt{1-\frac{v^2}{c^2}}} \qquad dt' = dt.\frac{\sqrt{\left(\sin\gamma - \frac{v.u_x}{c^2}\right)^2 + \left(\cos\gamma - \frac{v.u_y}{c^2}\right)^2}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{27}$$

Namely, $$d\mathbf{r}' = \frac{d\mathbf{r} - v.d\mathbf{t}}{\sqrt{\mathbf{1} - \frac{v^2}{c^2}}}; \qquad d\mathbf{t}' = \frac{d\mathbf{t} - \frac{v}{c^2} d\mathbf{r}}{\sqrt{\mathbf{1} - \frac{v^2}{c^2}}} \qquad \mathbf{u}' = \frac{d\mathbf{r}'}{dt'} \qquad (28)$$

These results show that the structure of differential VLT for curvilinear is the same previously viewed for rectilinear movement. Obviously, all this indicates that application of LLT to curvilinear movement is also valid, which will allow us to continue developing Relativity within only one theory.

The previous analysis was presented for the particular case at the beginning of measurements to differentiate the lengths of the differentials *dx* and *dx'*. The same situation can be displayed for the generic point P that the pulse of light is drawing in the space, where the same relationships are Also valid (see **Fig. 5).** Namely:

$$x' = \frac{x - \int v.dt.\sin\gamma}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \Rightarrow \quad dx' = \frac{dx - v.dt.\sin\gamma}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = \frac{y - \int v.dt.\cos\gamma}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \Rightarrow \quad dy' = \frac{dy - v.dt.\cos\gamma}{\sqrt{1 - \frac{v^2}{c^2}}}$$

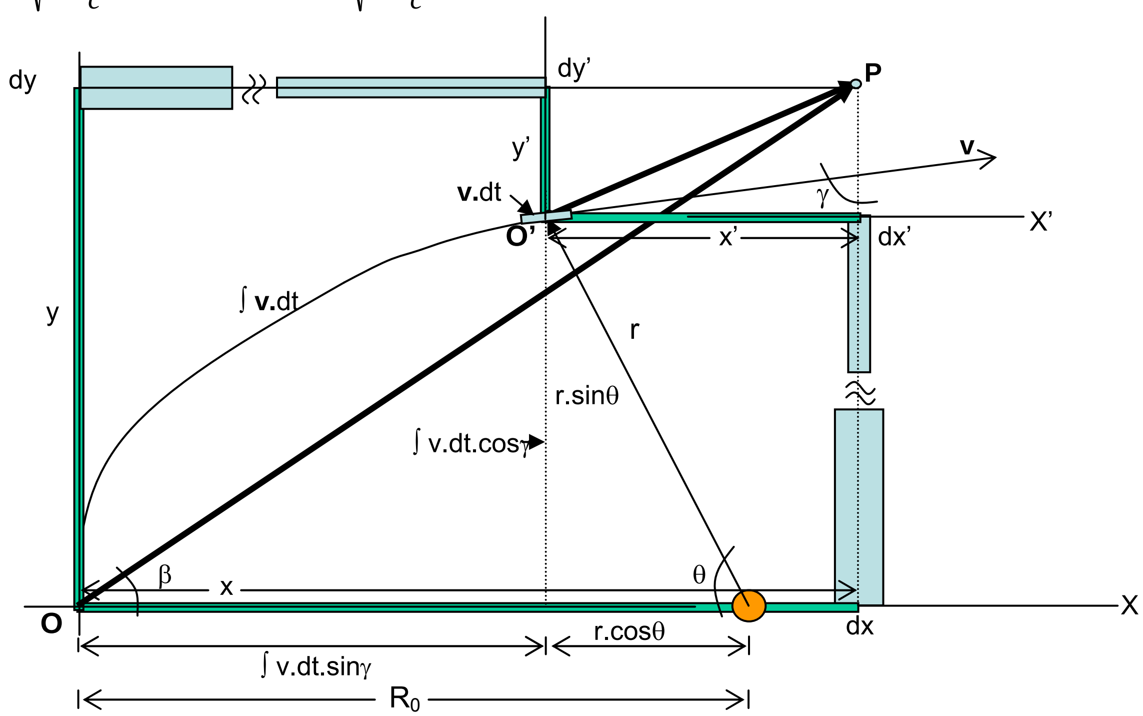

Fig. 5

Let's continue obtaining other local transformations, for instance, that for angle between inertial systems. This magnitude emanate from the relation between curvilinear length of arc s and length of radius R. Because both magnitudes are lengths, Lorentz factors cancel out, and angle becomes invariant to LLT (this result is different to that of explained by Einstein in SRT [1]):

$$\alpha' = \frac{s'}{R'} = \frac{\dfrac{s}{\sqrt{1-\dfrac{v^2}{c^2}}}}{\dfrac{R}{\sqrt{1-\dfrac{v^2}{c^2}}}} = \frac{s}{R} \;\Rightarrow\; \alpha' = \alpha; \qquad d\alpha' = \frac{ds'}{R'} = \frac{\dfrac{ds}{\sqrt{1-\dfrac{v^2}{c^2}}}}{\dfrac{R}{\sqrt{1-\dfrac{v^2}{c^2}}}} = \frac{ds}{R} \;\Rightarrow\; d\alpha' = d\alpha \qquad (29)$$

In this way, angular velocity transforms as:

$$\omega' = \frac{d\alpha'}{dt'} = \frac{d\alpha}{dt.\sqrt{1-\dfrac{v^2}{c^2}}} = \frac{\dfrac{d\alpha}{dt}}{\sqrt{1-\dfrac{v^2}{c^2}}} \qquad \Rightarrow \qquad \omega' = \frac{\omega}{\sqrt{1-\dfrac{v^2}{c^2}}} \qquad (30)$$

In next section we will obtain the LLT of Force and other physical magnitudes.

V LOCAL LORENTZ TRANSFORMATION OF FORCE, MASS AND OTHER DYNAMIC MAGNITUDES

Let's try to obtain a dynamical transformation for Force, based on already known LLT of magnitudes.

Let's suppose two masses, m_1 and m_2, rotating circularly around the center of mass CM of the system of the two masses, see **Fig. 6**. They additionally must move such that their centers of masses are always on a line passing through the center of mass of the system of the two masses, in order to ensure they move at the same angular velocity ω.

Because we have forced the masses to describe circular trajectories, it will allow us to remove gravitational forces from the analysis, i.e., only centrifugal forces will be considered. Let's suppose a Hercules, located at the center of mass CM, fixed, sustaining each mass through strong cords with each arm. Let there be three observers: Hercules at CM, as a fixed reference; observer 1, first moving reference on mass m_1 at a cord-distance r_1 from CM, and observer 2, second moving reference on mass m_2 at a cord-distance r_2 from CM.

1) As a first conclusion, for Hercules to be in equilibrium, he must measure equal and opposite tensions in each arm. Thus: $m_1.\omega^2.r_1 = m_2.\omega^2.r_2$.

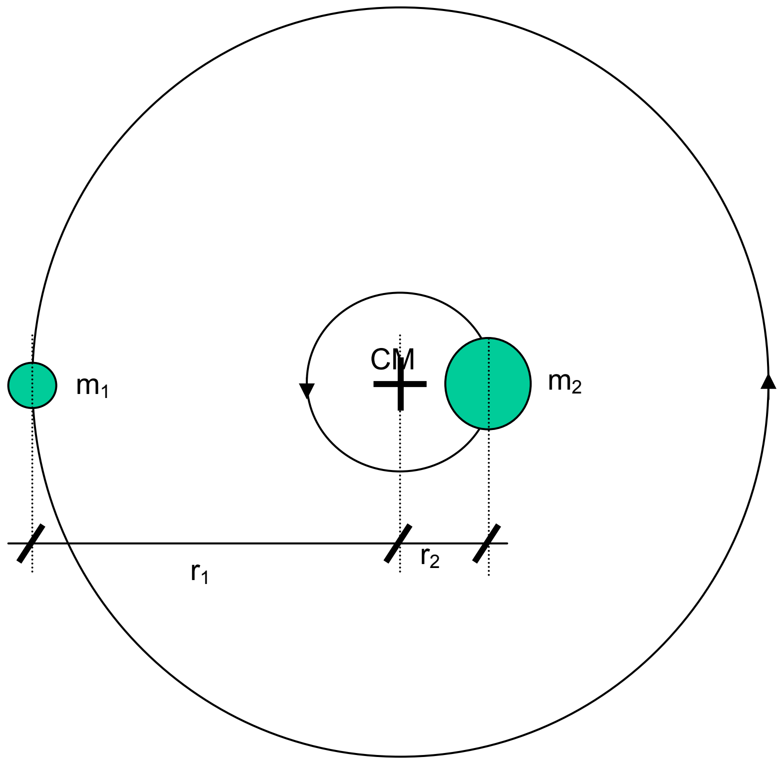

Fig. 5 Two masses rotating around a fixed center C

2) The tension T_1 exerted at one of Hercules' arm by cord r_1, measured by observer 1 on m_1, by taking point C as reference, will be $m'_1.\omega'^2.r'_1$, and tension T_2 exerted at Hercules' other arm by the cord r_2, measured by observer 2, by taking also the point C as reference, on m_2, will be $m''_2.\omega''^2.r''_2$. Let's <u>*assume*</u> that tensions T_1 and T_2 are equal, in order to maintain, as before, Hercules in equilibrium, which could also mean that Force would be invariant under LLT measurements. The values of physical magnitudes of moving masses involved in both tensions transform within the equations according to known LLT, with respect to what is measured by Hercules, in the following manner (except for masses, whose transformation is unknown):

$$m'_1.\omega'^2.r'_1 = m'_1.\frac{\omega^2}{\left(1-\frac{v_1^2}{c^2}\right)}.\frac{r_1}{\sqrt{1-\frac{v_1^2}{c^2}}} \equiv m''_2.\omega''^2.r''_2 = m''_2.\frac{\omega^2}{\left(1-\frac{v_2^2}{c^2}\right)}.\frac{r_2}{\sqrt{1-\frac{v_2^2}{c^2}}}$$

The only way for this relationship to always be consistent for any values of v_1 and v_2 is that masses have the following LLT:

$$m'_1 = \left(1-\frac{v_1^2}{c^2}\right)^{\frac{3}{2}}.m_1 \quad \text{and} \quad m''_2 = \left(1-\frac{v_2^2}{c^2}\right)^{\frac{3}{2}}.m_2 \qquad (30)$$

In this way Lorentz factors cancel out and this would imply that: $m_1.\omega^2.r_1 = m_2.\omega^2.r_2$, But, as this equality was previously correctly concluded in 1), then our assumption that tensions T_1 and T_2 were equal, is correct. This can be seen in another way. For maintaining Hercules in equilibrium (first conclusion), then tensions T_1 and T_2 must be equal. Thus, these results lead to that they are implied to each other, i.e.,

$$T_1 = m'_1.\omega'^2.r'_1 \equiv m_1.\omega^2.r_1 \equiv m_2.\omega^2.r_2 \equiv m''_2.\omega''^2.r''_2 = T_2.$$

Let's discuss in a deep way this equation. When observer 1 on m_1 (remember that he is fixed with respect to this mass, although the whole is a moving system) measures his mass, he measures m'_1, which is, for him, the rest mass, $m'_1 = M_{01}$. The same applies for the other observer 2 measuring the mass where he is on: $m''_2 = M_{02}$. So, given that through this special case of circular movement we have obtained the Lorentz factors for such masses in (30), and because the LLT of a magnitude always has the same structure, we can conclude, from equation (30), with the following strong statement: **In general, an inertial mass in movement at a velocity v is related to its rest mass in the following manner**:

$$m = \frac{M_0}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \tag{31}$$

This definition differs from the well-known Einstein's mass definition: $m = \frac{M_0}{\sqrt{1-\frac{v^2}{c^2}}}$. **In regards with this point, it's worth mentioning that Einstein also obtained equation (31) in his remarkable paper of 1905. He called this mass "longitudinal mass" [1], but later he discarded it from his work.**

Continuing the analysis by another route to obtain the relationship between mass and velocity, the following one is a more general way to arrive at the same result. For instance, let's consider a pair of masses, for instance the Sun and Earth as if they were the only bodies of the Solar System. Let's consider that as if the sun was the fixed reference (center of Sun is almost the center of mass of this system), and Earth moving around the Sun. Thus, Angular Momentum of Earth under LLT, measured by an observer from the Sun, is $m.r^2.\omega$ and its value must be constant, because there are no more forces acting around, and conservation of angular momentum holds. The "same" Angular Momentum of Earth, which moves following an elliptical path with variable velocity, measured by another observer, on Earth, taking Sun as his reference for measurements, is $m'.r'^2.\omega'$, which must also be constant, because the laws of nature are the same in any system of coordinates, becomes:

$$m'.r'^2.\omega' = m'.\frac{r^2}{\left(1-\frac{v^2}{c^2}\right)}.\frac{\omega}{\sqrt{1-\frac{v^2}{c^2}}} = CONSTANT \qquad (32)$$

Let's focus our attention on the explicit transformation of the elements involved within this last expression of angular momentum except for the earth mass, whose transformation is still considered unknown. By carefully observing the equation (32), we conclude that the only way for this expression being always constant, for any value of the variable v present in Lorentz factors in the denominator, is that the transformation for $m' = M_0$, cancels out the effect of such factors. For instance:

$$m' = M_0 = \left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}.m \qquad \Rightarrow \qquad m = \frac{M_0}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \qquad (33)$$

This is the same result previously obtained in equation (31). This means that Angular Momentum is invariant under LLT (and also the force). Given that Local Lorentz factors influence magnitude uniformly in all dimensions: We don't have different LLT for the same magnitude, contrasting to which is found in the Special Theory of Relativity (remember longitudinal or transversal expressions of mass, fields, etc).

VI. CONCLUSION

We have observed that Local Lorentz Transformations (LLT) give us the true dynamical value of a physical magnitude whose rest value is known, namely LLT inform us about the real dependence a physical magnitude has on the speed of light and on its own speed in space. In this way, the Theory of Relativity stops being a mysterious and complex subject, understood only by a few individuals, to become something simple and reasonable, familiar to anyone, and revealing to us a new and simple physical law that governs the movement of the bodies in space.

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