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The Lorentz Transformations: Correct Derivation and Consequences

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ABSTRACT: It is demonstrated that known assumptions of y and z coordinates transforming in a Galilean manner, used in the derivation of Lorentz Transformations are wrong. Such assumptions make Lorentz Transformations (LT) depend on the body's spatial orientation, i.e. the well-known transverse and longitudinal transformations, characterized by different scaling factors for the same magnitude. Development of Lorenz Transformations without these assumptions corrects their presentation. This mistake affects deeply Einstein's Special Theory of Relativity

KEYWORDS: Special Relativity, Galilean Transformations, Lorentz Transformations. Relativistic Time and Space

I. INTRODUCTION

After Einstein showed in 1905 the impossibility of establish whether an inertial system A moves relative to another one B or indeed the latter moves relative to A, without knowing any other reference [1], and settled down like a postulate, the constancy of the speed of light as a universal constant; and After H. A. Lorentz published his famous transformations in 1904 [2], many derivations of the Lorentz Transformations (LT) have been published in the relativistic literature. However, all of them, in some way, follow a procedure similar to that given by Lorentz, or by Einstein in cited publications, with some few exceptions, in the sense that all assume or "show" that y and z coordinates transform in a Galilean manner.

In this work, based on another previous one [3], the Lorentz Transformations are derived without making the above assumptions. In Section II Galilean Transformations are derived in a more general form, and Section III develops the derivation of Lorentz Transformations, presented as a modified Galilean Transformation through a factor in order to keep speed of light as a universal constant. Next section Galilean Transformations are developed in a general manner. In section III is presented a classical derivation of the LT **with** the mentioned assumptions, as it could be found in any modern text. In section IV the derivation of the mentioned transformations **without** the assumptions previously indicated is presented, preserving the constancy of light speed and the compatibility with the Maxwell equations doing the corresponding comparisons with the classic LT. Section V is dedicated to the vectorial derivation of the VLT and to some general demonstrations of its validity for any n-dimensional space. Section VI is dedicated to the conclusions of these results in the Physics.

II. GALILEAN TRANSFORMATIONS

Let's consider two parallel coordinate systems, with origins located at O and at O', which move, relative to each other with uniform translational motion along a line joining their origins, maintaining

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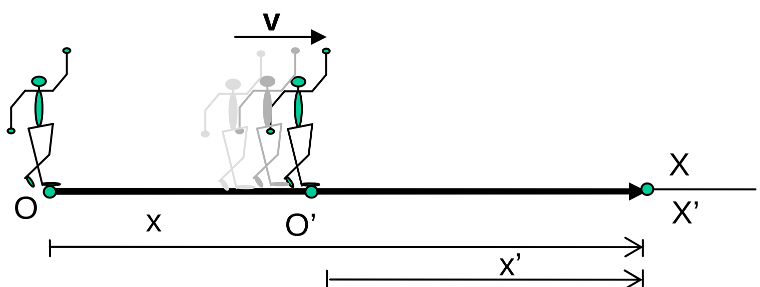

Fig. 1a

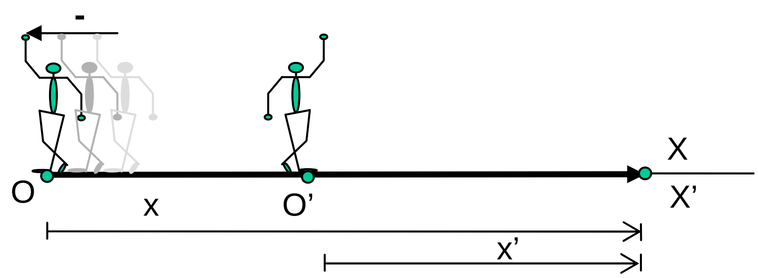

Fig. 1b

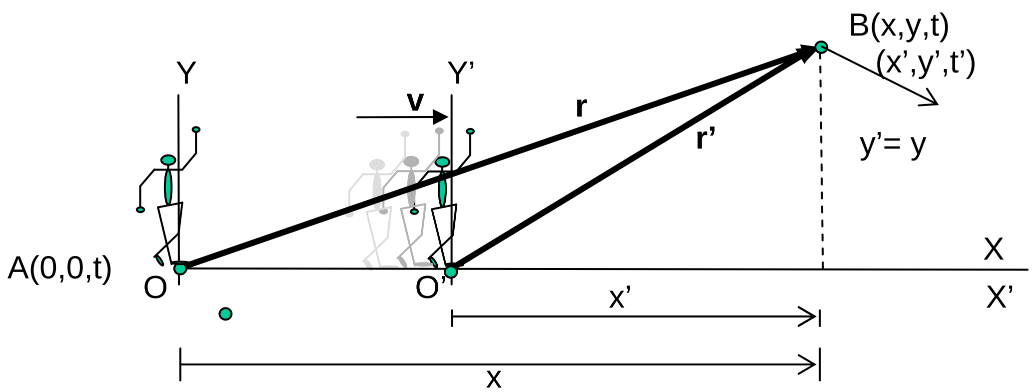

Fig. 2a

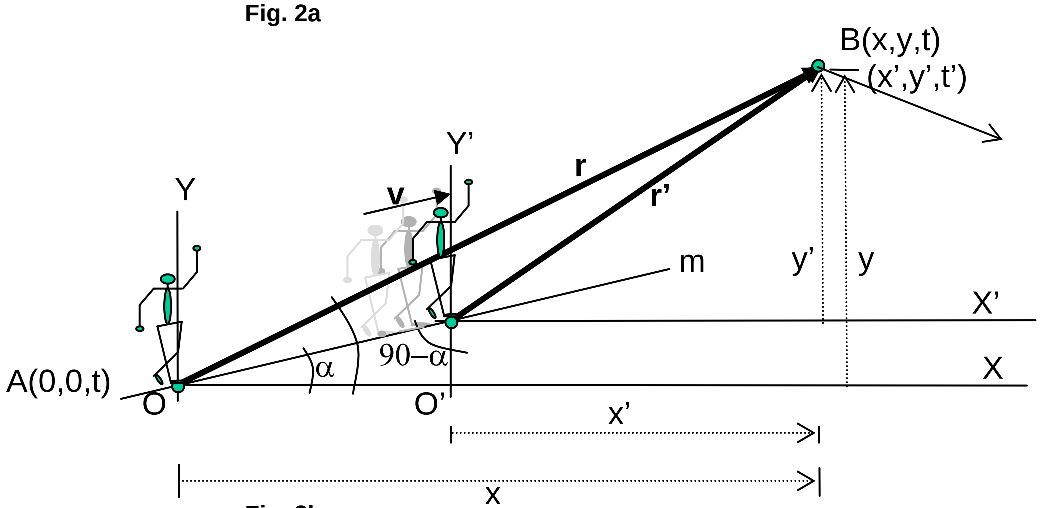

Fig. 2b

coordinates parallel. That is, the systems do not rotate relative to each other. In each origin is located an observer. During the relative motion of these two systems, observer at O sees observer at O' moving with speed v. On the contrary, observer at O' sees observer at O moving with speed $-v$.

In order to simplify, see Figs. 1a **and** 1b, referred to a one-dimensional motion. When (**Fig. 1a**) coming from $-\infty$, the origin O' of moving system coincides with fixed origin O, at $t = t' = 0$, and a bullet is shot parallel to X-axis, fixed observer at O measures a component x of the bullet displacement, but observer at origin O' of the moving system measures a distinct component x' of such displacement. Because in Galilean space time is the "same" for both observers, by observing **Fig. 1a** and from the "geometry of the problem", Galilean Transformations and its inverses relating measurements between both systems become:

Origin O, fixed and $t = t'$: (Direct LT)$_O$: $\begin{matrix} x' = x - v.t. \\ v_{x'} = v_x - v \end{matrix}$ (Inverse LT)$_O$: $\begin{matrix} x = x' + v.t \\ v_x = v_{x'} + v \end{matrix}$ (1)

Origin O', fixed and $t = t'$: (Direct LT)$_{O'}$: $\begin{matrix} x = x' + v.t. \\ v_x = v_{x'} + v \end{matrix}$ (Inverse LT)$_{O'}$: $\begin{matrix} x' = x - v.t \\ v_{x'} = v_x - v \end{matrix}$ (2)

As we observe, measurements of the speed of the bullet are different for both observers. This is a logic result according to common sense. Problem arises if the bullet is a pulse of light, because it leads to either an inconsistency or light speed does not preserve constant: $c = c' + v$ or $c' = c - v$. Observe also that direct transformations for O fixed are equal to that Inverse for O' fixed, See (1) (2).

For a two-dimensional Galilean space, we can start with the example referred to that of **Fig. 2a**. When O' coming from $-\infty$, coincides with fixed origin O, at $t = t' = 0$, and both observers are watching the flight of a plane overhead, fixed observer at O measures a component x and a component y of the plane displacement, but the moving observer at O' measures a component x' of such displacement equal to $x' = x - v.t$ and a component y' which is the "same" , "by symmetry", as that measure by observer at O. Again time is the "same" for the two observers and plane is moving nonparallel to some axis. Its speed-component on X-axis shall be $v_{x'} = v_x - v$. From the geometry of the problem, Galilean Transformations become:

Origin O, fixed and $t = t'$.: (Direct LT)$_O$: $\begin{matrix} x' = x - v.t. \\ y' = y \\ v_{x'} = v_x - v \\ v_{y'} = v_y \end{matrix}$ (Inverse LT)$_O$: $\begin{matrix} x = x' + v.t \\ y = y' \\ v_x = v_{x'} + v \\ v_y = v_{y'} \end{matrix}$ (3)

Origin O', fixed and $t = t'$.: (Direct LT)$_{O'}$: $\begin{matrix} x = x' + v.t \\ y = y'. \\ v_x = v_{x'} + v \\ v_y = v_{y'} \end{matrix}$ (Inverse LT)$_{O'}$: $\begin{matrix} x' = x - v.t \\ y' = y \\ v_{x'} = v_x - v \\ v_{y'} = v_y \end{matrix}$ (4)

The relationships (4) are valid for considering system O' as fixed and system O as the moving one.

In order to generalize the results for the two-dimensional case, let's represent this situation in a more general way. referring now to the **Fig. 2b**, where the two observers, with all the equipment for doing measurements of length, time and velocities onto moving projectiles; the first one located at the origin of coordinates of fixed system O, and the second one located at the origin of coordinates of a system O' moving at a constant velocity v , relative to O, but now, **along an inclined line** m joining both origins. The inclined line forms an angle α with the X axis. The axes of both systems maintain always parallel, say X parallel to X', Y parallel to Y' and Z parallel to Z' but they will not coincide anymore. Let $\mathbf{r}$ be the radio-vector of a rectilinear trajectory of a plane flying non-parallel to some axis, measured by the observer at O and Let $\mathbf{r}'$ be the radio-vector of the rectilinear trajectory of the plane, measured by the observer at O'. The goal will be to obtain relationships between the observers' measurements, such that they must be valid for any velocity of the plane (or for any projectile). In order to arrive at similar relationships we will apply our common sense, and in this way we will have the general expression of Galilean Transformations for a two-dimensional space:

Origin O, fixed and $t = t'$.: (Direct LT)$_O$:
$$\begin{aligned} x' &= x - t.v.cos\,\alpha \\ y' &= y - t.v.sin\,\alpha \\ v_{x'} &= v_x - v.cos\,\alpha \\ v_{y'} &= v_y - v.sin\,\alpha \end{aligned}$$
(Inverse LT)$_O$:
$$\begin{aligned} x &= x' + t.v.cos\,\alpha \\ y &= y' + t.v.sin\,\alpha \\ v_x &= v_{x'} + v.cos\,\alpha \\ v_y &= v_{y'} + v.sin\,\alpha \end{aligned} \quad (5)$$

Origin O', fixed and $t = t'$.: (Direct LT)$_{O'}$:
$$\begin{aligned} x &= x' + t.v.cos\,\alpha \\ y &= y' + t.v.sin\,\alpha \\ v_x &= v_{x'} + v.cos\,\alpha \\ v_y &= v_{y'} + v.sin\,\alpha \end{aligned}$$
(Inverse LT)$_{O'}$:
$$\begin{aligned} x' &= x - t.v.cos\,\alpha \\ y' &= y - t.v.sin\,\alpha \\ v_{x'} &= v_x - v.cos\,\alpha \\ v_{y'} &= v_y - v.sin\,\alpha \end{aligned} \quad (6)$$

The relationships (6) are valid for considering system O' as fixed and system O as the moving one.

Three-dimensional case: see **Fig. 3a**, which is the most used model in the derivation of Galilean Transformations, may be because it easily allows taking the Galilean assumptions, as it will be observed when we arrive at the Lorentz Transformations classical derivation.

In this configuration, both X and X' axes coincide along the line of the relative motion and the YZ and Y'Z' axes parallel to each other. When coming from $-\infty$, the origin O' of moving system coincides with fixed origin O, at $t = t' = 0$, and both observers are watching the flight of a plane non-parallel to some of the axes, fixed observer at O measures a component x of the plane displacement, but the moving observer at O' measures a component x' of such displacement equal to $x' = x - v.t$. Because of the parallelism of the YZ and Y'Z' axes, the value of the coordinates are the same for both observers, $y = y'$ and $z = z'$, and time is the "same" for both observers, $t = t'$. From the geometry of the problem, Galilean Transformations for this particular case become:

Origin O, fixed and $t = t'$.: (Direct LT)$_O$:
$$\begin{aligned} &x' = x - t.v.cos\,\alpha \\ &y' = y; \qquad z' = z \\ &v_{x'} = v_x - v.cos\,\alpha \\ &v_{y'} = v_y; \quad v_{z'} = v_z \end{aligned}$$
(Inverse LT)$_O$:
$$\begin{aligned} &x = x' + t.v.cos\,\alpha \\ &y = y'; \qquad z = z' \\ &v_x = v_{x'} + v.cos\,\alpha \\ &v_y = v_{y'}; \quad v_z = v_{z'} \end{aligned} \quad (7)$$

Origin O', fixed and $t = t'$: (Direct LT)$_{O'}$:
$$\begin{aligned} x &= x' + t.v.\cos\alpha \\ y &= y'; \qquad z = z' \\ v_x &= v_{x'} + v.\cos\alpha \\ v_y &= v_{y'}; \qquad v_z = v_{z'} \end{aligned}$$
(Inverse LT)$_{O'}$:
$$\begin{aligned} x' &= x - t.v.\cos\alpha \\ y' &= y; \qquad z' = z \\ v_{x'} &= v_x - v.\cos\alpha \\ v_{y'} &= v_y; \qquad v_{z'} = v_z \end{aligned} \qquad (8)$$

The relationships (8) are valid for considering system O' as fixed and system O as the moving one.

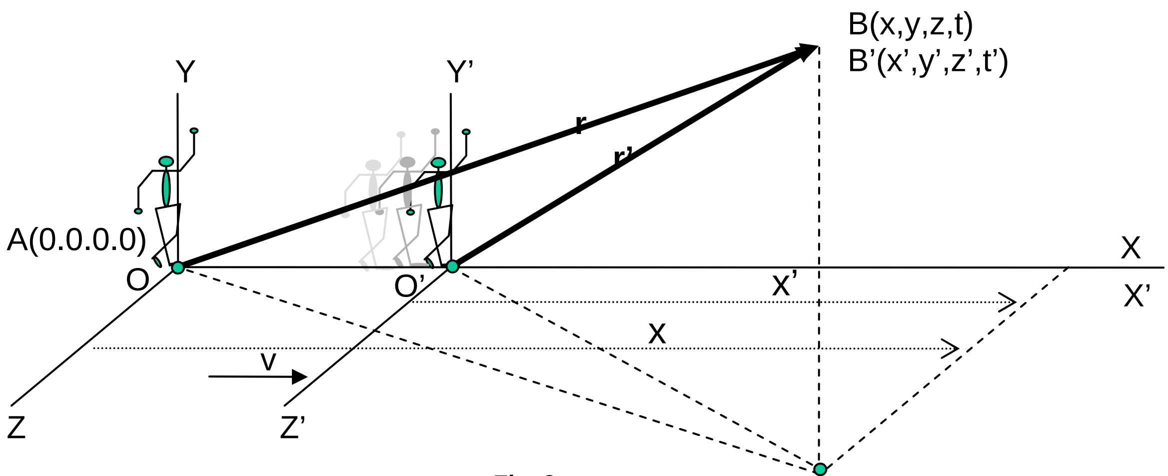

Fig. 3a

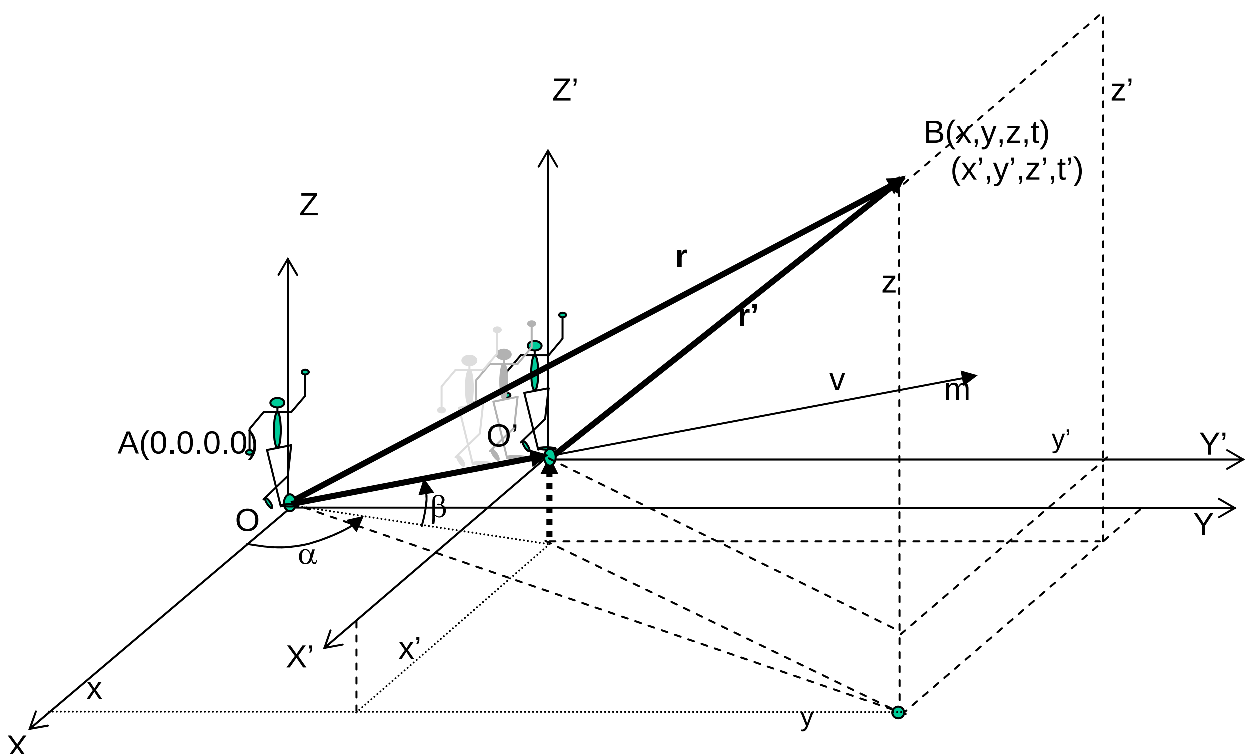

Fig. 3b

Now, let's represent this three-dimensional situation in a more general way. referring to the **Fig. 3b**, where the two observers, with all the equipment for doing measurements of length, time and velocities onto moving projectiles; the first one located at the origin of coordinates of a fixed system O, the same as before, and the second one located at the origin of coordinates of a moving system O'. System O' moves at a constant velocity v, relative to O, But now, **along an inclined line** m, such that both observers are on the same line. The inclined line forms an angle β with the plane XY and its projection on this plane XY an angle α with the X-axis.

The axes of both systems maintain always parallel, say X parallel to X', Y parallel to Y' and Z parallel to Z', but they are never coincident. Let $\mathbf{r}$ be the radio-vector of a plane in rectilinear (also inclined) flight, measured by the observer at O and Let $\mathbf{r}'$ be the radio-vector of the same plane, measured by the observer at O'. In this way, applying our known common sense, we will have the general expression of Galilean Transformations for a three-dimensional space:

O, fixed and $t = t'$: (Direct LT)$_O$:

$$\begin{aligned} x' &= x - t.v.\cos\alpha.\cos\beta \\ y' &= y - t.v.\sin\alpha.\cos\beta \\ z' &= z - t.v.\sin\beta \\ v_{x'} &= v_x - v.\cos\alpha.\cos\beta \\ v_{y'} &= v_y - v.\sin\alpha.\cos\beta \\ v_{z'} &= v_z - v.\sin\beta \end{aligned}$$

(Inverse LT)$_O$:

$$\begin{aligned} x &= x' + t.v.\cos\alpha.\cos\beta \\ y &= y' + t.v.\sin\alpha.\cos\beta \\ z &= z' + t.v.\sin\beta \\ v_x &= v_{x'} + v.\cos\alpha.\cos\beta \\ v_y &= v_{y'} + v.\sin\alpha.\cos\beta \\ v_z &= v_{z'} + v.\sin\beta \end{aligned} \qquad (9)$$

O', fixed and $t = t'$: (Direct LT)$_{O'}$:

$$\begin{aligned} x &= x' + t.v.\cos\alpha.\cos\beta \\ y &= y' + t.v.\sin\alpha.\cos\beta \\ z &= z' + t.v.\sin\beta \\ v_x &= v_{x'} + v.\cos\alpha.\cos\beta \\ v_y &= v_{y'} + v.\sin\alpha.\cos\beta \\ v_z &= v_{z'} + v.\sin\beta \end{aligned}$$

(Inverse LT)$_{O'}$:

$$\begin{aligned} x' &= x - t.v.\cos\alpha.\cos\beta \\ y' &= y - t.v.\sin\alpha.\cos\beta \\ z' &= z - t.v.\sin\beta \\ v_{x'} &= v_x - v.\cos\alpha.\cos\beta \\ v_{y'} &= v_y - v.\sin\alpha.\cos\beta \\ v_{z'} &= v_z - v.\sin\beta \end{aligned} \qquad (10)$$

The relationships (10) are valid for considering system O' as fixed and system O as the moving one.

In next sections we will derive the Lorentz Transformations in the classical way and later correctly in order to compare both results and discuss why assuming, $y' = y$ and $z' = z$, is erroneous and how this erroneous assumptions influence results.

III. CLASSICAL DERIVATION OF LORENTZ TRANSFORMATIONS (LT)

As it is well known, the discussion of past centuries about a medium to transport light was undoubtedly clarified by Einstein in his seminal 2005-paper [1] with the conclusion of the nonexistence of Ether. I agree his argument about this theme. This discussion comes out because Galilean Transformations were not compatible with Maxwell's Equations. For example, in agreement with the Galilean analysis, an observer who moves at constant speed v, in the same sense of the speed of a pulse of light c sent by the lantern of a fixed observer, will deduce that the speed of the light pulse is $c-v$. But, if one moves contrary he will deduce that the pulse moves away to a speed $c+v$. That is to say, for a Galilean analysis the speed of a light pulse depends on the observer. In 1881 the experiment of Michelson and Morley was made to measure the speed of light in different directions with the clear and surprising result that the speed of light was the same in all directions. As an explanation to this result the Transformations of Lorentz (LT) arose.

For deriving LT the following postulates or assumptions are needed:

1) ***Principle of Relativity***: “Physical laws should be the same in all inertial reference frames”. This principle was first explicitly enunciated by Galileo Galilei in 1639 in his “*Dialogue Concerning the Two Chief World Systems*”. Now it is known as the Einstein‘s relativity principle. It is a universal accepted postulate.

2) ***Invariance of Light Speed***: “The speed of light in vacuum is the same for all inertial observers, independent of the speed of the emitting body”. The past two centuries, and in current one, this universal constant has been checked until the following current exact value: 299,792,458 meters per second.

Majority of authors use the particular configuration of **Fig. 2a** (two dimensions) or **Fig. 3a** (three dimensions) which are not general configurations of relative motion. Also, due to the line of relative motion joining the origin of fixed system O with the origin of the moving system O’ is simultaneously perpendicular to the Y and/or Z axes and to the Y’ and/or Z’ axes, “common sense” and/or the “geometry or symmetry of the problem” ***indicate*** that the displacements parallel to these axes seen by observers in one and in the other system are the “same”, then: $y'=y$ and $z'=z$. In this sense, we will present a simple and direct procedure that resumes the different methods used by other authors since 1904 to the present time, with such assumptions.

Lorentz Transformations must be valid also if projectile is a pulse of light. In such case, for preserving the constancy of the speed of light, what is done mathematically is that a factor k, to be determined, is introduced into the Galilean Transformations. Remember that projectile or the light pulse is sent when both origins coincide at $t'=t=0$. Since time measured by observers could not be the same, we may assume, in order to ease calculations that time measured observers are related as $t'=k.(t-a.x)$, where a is another factor to be determined (for reducing to Galilean Transformation $k=1$ and $a=0$). Thus, the following first set of relationships referred to **Fig. 2a**, similar to those in (3), are established:

$$x'=k.(x-v.t) \tag{11}$$

$$y'=y \tag{12}$$

$$t'=k.(t-a.x) \tag{13}$$

Equation (11) is obtained from **Fig. 2a** by considering that system O is fixed and for moving system O’ coming from $-\infty$. Observe that the expression (12), is not derived from equations but from “common sense”. So, let’s obtain the value of k, by taking the projectile being a pulse of light. In such that case, the speed of the light pulse, c, will be the same measured by both observers:

$$x^2 + y^2 = c^2.t^2 \tag{14}$$
$$x'^2 + y'^2 = c^2.t'^2 \tag{15}$$

Making substitutions of expressions (11), (12) and (13) in (15), we have:

$k^2.(x^2 - 2.v.x.t. + v^2.t^2) + y^2 = c^2.k^2.(t^2 - 2.a.x.t + a^2.x^2)$, or in an ordered manner:

$$x^2.(k^2 - c^2.k^2.a^2) + 2.x.t.(k^2.v - a.k^2.c^2) + y^2 = c^2.t^2.\left(k^2 - k^2.\frac{v^2}{c^2}\right) \tag{16}$$

This result must be identical to equation (14). So,

$$\begin{aligned} k^2 - c^2.k^2.a^2 &= 1 \\ k^2.v - a.k^2.c^2 &= 0 \\ k^2 - k^2.\frac{v^2}{c^2} &= 1 \end{aligned} \quad \Rightarrow \quad \begin{aligned} k &= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \\ a &= \frac{v}{c^2} \end{aligned} \tag{17}$$

Thus, we have finally the set of Lorentz Transformations for two dimensions, as they have been known for more than one hundred years, for k having the value given in (17):

Origin O, fixed and $t = t'$: (Direct LT)$_O$: (Inverse LT)$_O$:

$$\begin{aligned} x' &= k(x - v.t) \\ y' &= y \\ t' &= k\left(t - \frac{v}{c^2}.x\right) \\ u_{x'} &= \frac{(u_x - v)}{1 - \frac{v}{c^2}.u_x} \\ v_{y'} &= \frac{u_y}{k.\left(1 - \frac{v}{c^2}u_x\right)} \end{aligned} \qquad \begin{aligned} x &= k(x' + v.t') \\ y &= y' \\ t &= k\left(t' + \frac{v}{c^2}.x'\right) \\ u_x &= \frac{(u_{x'} + v)}{1 + \frac{v}{c^2}.u_{x'}} \\ v_{y'} &= \frac{u_{y'}}{k.\left(1 + \frac{v}{c^2}.u_{x'}\right)} \end{aligned} \tag{18}$$

Observe that the Inverse Transformations are obtained from Direct LT. In fact:

$$x' = k.(x - v.t) \Rightarrow x = \frac{x'}{k} + v.t \Rightarrow x = \frac{x'}{k} + v.\left(\frac{t'}{k} + \frac{v}{c^2}.x\right) \Rightarrow x = \frac{x' + v.t'}{k.(1 - v^2/c^2)} \Rightarrow x = k.(x' + v.t')$$

$$t' = k.\left(t - \frac{v}{c^2}.x\right) \Rightarrow t = \frac{t'}{k} + \frac{v}{c^2}.x \Rightarrow t = \frac{t'}{k} + \frac{v}{c^2}.\left(\frac{x'}{k} + v.t\right) \Rightarrow t = \frac{t' + \frac{v}{c^2}x'}{k.(1 - v^2/c^2)} \Rightarrow t = k.\left(t' + \frac{v}{c^2}\right) \tag{19}$$

Changing the role of observers:

Origin O, fixed and $t = t'$: (Direct LT)$_O$:

$$\begin{aligned} x &= k(x' + v.t') \\ y &= y' \\ t &= k\left(t' + \frac{v}{c^2}.x'\right) \\ u_x &= \frac{(u_{x'} + v)}{1 + \frac{v}{c^2}.u_{x'}} \\ v_{y'} &= \frac{u_{y'}}{k.\left(1 + \frac{v}{c^2}.u_{x'}\right)} \end{aligned}$$

(Inverse LT)$_O$:

$$\begin{aligned} x' &= k(x - v.t) \\ y' &= y \\ t' &= k\left(t - \frac{v}{c^2}.x\right) \\ u_{x'} &= \frac{(u_x - v)}{1 - \frac{v}{c^2}.u_x} \\ v_{y'} &= \frac{u_y}{k.\left(1 - \frac{v}{c^2}u_x\right)} \end{aligned} \qquad (20)$$

Inverse Transformations in (19) and direct ones in (20) and conversely, although they have the same symbols they refer to distinct situations.

Note 1: Another reason to take as so the relationship (12) is because if it were established for O fixed and a moving O' the logic statement would be that $y' = k.y$. But if O' is the one considered fixed and a moving system O, the logic statement should be $y = k.y'$. This would imply $k = 1$, and this way brings us back to Galilean Transformations (!). Thus, the only "solution" *was to assume directly,* $y' = y$.

Note 2: Furthermore, in author's opinion the main reason for erroneously assuming $y' = y$ and $z' = z$ was the choosing as basic configuration of relative motion for deriving LT that with the coincidence of X and X' axes as the starting point. This can be observed, virtually, in any publication touching this theme.

The procedure for obtaining the set for three dimensions from **Fig. 3a** is similar to the previous one, obtaining the same solutions and one more assumption: $z' = z$.

Origin O, fixed and $t = t'$: (Direct LT)$_O$:

$$\begin{aligned} x' &= k(x - v.t) \\ y' &= y; \quad z' = z \\ t' &= k\left(t - \frac{v}{c^2}.x\right) \\ u_{x'} &= \frac{(u_x - v)}{1 - \frac{v}{c^2}.u_x} \\ u_{y'} &= \frac{u_y}{k.\left(1 - \frac{v}{c^2}u_x\right)} \\ u_{z'} &= \frac{u_z}{k.\left(1 - \frac{v}{c^2}u_x\right)} \end{aligned}$$

(Inverse LT)$_O$:

$$\begin{aligned} x &= k(x' + v.t') \\ y &= y' \\ t &= k\left(t' + \frac{v}{c^2}.x'\right) \\ u_x &= \frac{(u_{x'} + v)}{1 + \frac{v}{c^2}.u_{x'}} \\ v_y &= \frac{u_{y'}}{k.\left(1 + \frac{v}{c^2}.u_{x'}\right)} \\ u_z &= \frac{u_z}{k.\left(1 - \frac{v}{c^2}u_{x'}\right)} \end{aligned} \qquad (21)$$

Origin O, fixed and $t = t'$: (Direct LT)$_O$:

$$x' = k(x - v.t)$$
$$y' = y; \quad z' = z$$
$$t' = k\left(t - \frac{v}{c^2}.x\right)$$
$$u_{x'} = \frac{(u_x - v)}{1 - \frac{v}{c^2}.u_x}$$
$$u_{y'} = \frac{u_y}{k.\left(1 - \frac{v}{c^2}u_x\right)}$$
$$u_{z'} = \frac{u_z}{k.\left(1 - \frac{v}{c^2}u_x\right)}$$

(Inverse LT)$_O$:

$$x = k(x' + v.t')$$
$$y = y'$$
$$t = k\left(t' + \frac{v}{c^2}.x'\right)$$
$$u_x = \frac{(u_{x'} + v)}{1 + \frac{v}{c^2}.u_{x'}}$$
$$v_y = \frac{u_{y'}}{k.\left(1 + \frac{v}{c^2}.u_{x'}\right)}$$
$$u_z = \frac{u_z}{k.\left(1 - \frac{v}{c^2}u_{x'}\right)} \qquad (22)$$

IV. CORRECT DERIVATION OF LORENTZ TRANSFORMATIONS (1)

Instead of using the particular examples in **Fig. 2a** and **Fig. 3a** we will use the general configurations in **Figs. 2b** and **3b** as the correct starting point for deriving LT. We follow the same sequence of presentation used before for two and three dimensions.

Referring to **Fig. 2b,** referred to relative motion in two dimensions, when origin O', moving along an inclined line m at constant speed v and coming from the left side of m, coincides with the fixed origin O, a light pulse is sent in any direction. By defining α, as the angle between line of relative motion m, and X axis, the following equations hold:

$$x^2 + y^2 = c^2.t^2 \qquad \text{for,} \qquad x' = k.(x - v.\cos\alpha.t)$$
$$x'^2 + y'^2 = c^2.t'^2 \qquad\qquad y' = k.(y - v.t.\sin\alpha) \qquad (23)$$

Based on these relationships, by substituting, working on and grouping properly, we obtain:

$$c^2.t'^2 = k^2.[(x - v.\cos\alpha.t)^2 + (y - v.t.\sin\alpha)^2]$$
$$c^2.t'^2 = k^2.\left[(x^2 + y^2) + v^2.t^2(\cos^2\alpha + \sin^2\alpha) - 2.v.x.(t.\cos\alpha) - 2.v.y.(t.\sin\alpha)\right] \qquad (24)$$
$$c^2.t'^2 = k^2.\left[c^2.t^2 + v^2.t^2\right] - 2.v.x.(t.\cos\alpha) - 2.v.y.(t.\sin\alpha)$$

Substituting: $c^2.t^2 \equiv c^2.t^2.(\sin^2\alpha + \cos^2\alpha)$, and $v^2.t^2 \equiv v^2.\frac{x^2 + y^2}{c^2}$; and grouping, we get:

$$c^2.t'^2 = k^2.\{[c^2.(t.\cos\alpha)^2 - 2.v.x.(t.\cos\alpha) + v^2.\frac{x^2}{c^2}] + [c^2.(t.\sin\alpha)^2 - 2.v.y.(t.\sin\alpha) + v^2.\frac{y^2}{c^2}]\}$$

$$c^2.t'^2 = k^2.[(c.t.\cos\alpha - \frac{v}{c}.x)^2 + (c.t.\sin\alpha - \frac{v}{c}.y)^2] = c^2.k^2.[(t.\cos\alpha - \frac{v}{c^2}.x)^2 + (t.\sin\alpha - \frac{v}{c^2}.y)^2]$$

Without assuming anything and dividing by c^2 the last relationship, it is obtained the following expression for time:

$$t'^2 = k^2.[(t.\cos\alpha - \frac{v}{c^2}.x)^2 + (t.\sin\alpha - \frac{v}{c^2}.y)^2] \quad (25)$$

Before continuing let's obtain the value of k, from previous equations, by the forming the identity:

$$c^2.k^2.\left[\left(t.cos\,\alpha - \frac{v}{c^2}.x\right)^2 + \left(t.sin\,\alpha - \frac{v}{c^2}.y\right)^2\right] \equiv$$
$$\equiv k^2.\left[\left(x^2+y^2\right) + v^2.t^2\left(cos^2\,\alpha + sin^2\,\alpha\right) - 2.v.x.(t.cos\,\alpha) - 2.v.y.(t.sin\,\alpha)\right]$$

Simplifying and reordering we have: $\left(c^2.k^2 - v^2.k^2\right).t^2 = \left(k^2 - \frac{k^2.v^2}{c^2}\right).x^2 + \left(k^2 - \frac{k^2.v^2}{c^2}\right).y^2$

This result must be identical to: $c^2.t^2 = x^2 + y^2$, therefore:

$$c^2.k^2 - v^2.k^2 = c^2$$
$$k^2 - \frac{k^2.v^2}{c^2} = 1 \qquad \text{Solving, we have:} \qquad k^2 = \frac{1}{1-\frac{v^2}{c^2}} \quad \Rightarrow \quad k = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad (26)$$

Let's continue. By observing carefully the right hand side of expression (25), it takes us to the definition of the magnitude of time by considering it as a vector. Thus, as it is suggested, the previous expression can be re-organized into its corresponding two-dimensional vectorial structure, in the following way:

$$\mathbf{t'} = k.\left[\left(t.\cos\alpha - \frac{v}{c^2}.x.\right)\mathbf{i} + \left(t.sin\alpha - \frac{v}{c^2}.y\right).\mathbf{j}\right] = k.\left[t.\cos\alpha.\mathbf{i} - \frac{v}{c^2}.x.\mathbf{i} + t.sin\alpha.\mathbf{j} - \frac{v}{c^2}.y.\mathbf{j}\right]$$
$$\mathbf{t'} = k.\left[\left(t.\cos\alpha.\mathbf{i} + t.sin\alpha.\mathbf{j}\right) - \frac{v}{c^2}.(x.\mathbf{i} + y.\mathbf{j})\right] = k.\left[\left(t_x.\mathbf{i} + t_y.\mathbf{j}\right) - \frac{v}{c^2}.(x.\mathbf{i} + y.\mathbf{j})\right] \quad (27)$$

Thus, defining: $\begin{Bmatrix} t_x = t.\cos\alpha \\ t_y = t.\sin\alpha \\ \mathbf{t} = t_x\mathbf{i} + t_y\mathbf{j} \end{Bmatrix}$ and $\begin{Bmatrix} t'_x = k.\left(t_x - \frac{v}{c^2}.x\right) \\ t'_y = k.\left(t_y - \frac{v}{c^2}.y\right) \end{Bmatrix} \Rightarrow \begin{Bmatrix} \mathbf{t'} = k.\left(\mathbf{t} - \frac{v}{c^2}.\mathbf{r}\right) \\ \mathbf{r'} = k.(\mathbf{r} - v.\mathbf{t}) \end{Bmatrix}$ (28)

It can be realized that this vector structure of time can be easily obtained for any number of dimensions by repeating this same procedure. For instance, in the three-dimensional case, see the **Fig. 3b**; the following general relationships can be constructed from the geometry of the problem:

$$x^2 + y^2 + z^2 = c^2.t^2 \qquad x' = k.(x - v.t.\cos\alpha.\cos\beta) \qquad t_x = t.\cos\alpha.\cos\beta$$
$$x'^2 + y'^2 + z'^2 = c^2.t'^2 \qquad y' = k.(y - v.t.\sin\alpha.\cos\beta) \quad \text{and by defining:} \quad t_y = t.\sin\alpha.\cos\beta \qquad (29)$$
$$z' = k.(z - v.t.\sin\beta) \qquad t_z = t\sin\beta$$

From relationships in (29) it can be obtained again (following a similar procedure to that previously used) the already familiar vector structure expression of time, for three (or for any number of) dimensions:

$$t'^2 = k^2.[(t_x - \frac{v}{c^2}.x)^2 + (t_y - \frac{v}{c^2}.y)^2 + (t_z - \frac{v}{c^2}.z)^2] \quad \Rightarrow \quad \begin{matrix} \mathbf{t}' = k.(\mathbf{t} - \frac{v}{c^2}.\mathbf{r}) \\ \mathbf{r}' = k.(\mathbf{r} - v.\mathbf{t}) \end{matrix} \qquad (30)$$

All these results lead consistently to consider the behavior of time as a vector when it is referred to observers located in inertial systems with relative motion. Let's find the expression of $\mathbf{t}$ and $\mathbf{r}$ in function of $\mathbf{t}'$ and $\mathbf{r}'$ or their inverse transformations by manipulating the expressions of VLT in (30):

$$\mathbf{r}' = k.(\mathbf{r} - v.\mathbf{t}) \quad \Rightarrow \quad \mathbf{r} = \frac{\mathbf{r}'}{k} + v.\mathbf{t} \quad \Rightarrow \quad \mathbf{r} = \frac{\mathbf{r}'}{k} + v.\left(\frac{\mathbf{t}'}{k} + \frac{v}{c^2}.\mathbf{r}\right) \quad \Rightarrow \quad \mathbf{r} = \frac{\mathbf{r}' + v.\mathbf{t}'}{k.\left(1 - \frac{v^2}{c^2}\right)}$$

$$\mathbf{t}' = k.\left(\mathbf{t} - \frac{v}{c^2}.\mathbf{r}\right) \quad \Rightarrow \quad \mathbf{t} = \frac{\mathbf{t}'}{k} + \frac{v}{c^2}.\mathbf{r} \quad \Rightarrow \quad \mathbf{t} = \frac{\mathbf{t}'}{k} + \frac{v}{c^2}.\left(\frac{\mathbf{r}'}{k} + v.\mathbf{t}\right) \quad \Rightarrow \quad \mathbf{t} = \frac{\mathbf{t}' + \frac{v}{c^2}\mathbf{r}'}{k.\left(1 - \frac{v^2}{c^2}\right)}$$

For $k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$, :we have the inverse transformations: $\left\{ \begin{matrix} \mathbf{r} = k.(\mathbf{r}' + v.\mathbf{t}') \\ \mathbf{t} = k.\left(\mathbf{t}' + \frac{v}{c^2}\mathbf{r}'\right) \end{matrix} \right\}$ (31)

Value of k is also easily obtained from the obtained expressions of the radio-vectors measured by each observer, considering system O fixed, and O' as the moving one, i.e.: $\mathbf{r}' = k.(\mathbf{r} - v.\mathbf{t})$ and $\mathbf{r} = k.(\mathbf{r}' + v.\mathbf{t}')$. In fact, by substituting $\mathbf{t}' = \frac{\mathbf{r}'}{c}$ and $\mathbf{t} = \frac{\mathbf{r}}{c}$ in both expressions we have:

$$\mathbf{r}' = k.\left(\mathbf{r} - v.\frac{\mathbf{r}}{c}\right) = k.\mathbf{r}.\left(1 - \frac{v}{c}\right)$$
$$\mathbf{r} = k.\left(\mathbf{r}' + v.\frac{\mathbf{r}'}{c}\right) = k.\mathbf{r}'.\left(1 + \frac{v}{c}\right)$$

And multiplying by each other we get:

$$\mathbf{r}'\bullet\mathbf{r} = k^2.\mathbf{r}\bullet\mathbf{r}'.\left(1-\frac{v^2}{c^2}\right) \qquad \Rightarrow \qquad k = \sqrt{\frac{1}{1-\frac{v^2}{c^2}}}$$

The invariance of the space-time interval of the Special Theory of Relativity, $c^2.t'^2 - r'^2 = c^2.t^2 - r^2$ is preserved for **any number of dimensions.** This means that this new presentation of LT is also valid not only for light but for any projectile, as it was expected. Demonstration follows. Substituting general coordinate components (n-dimensional) in the expression of the space-time interval, $c^2.t'^2 - r'^2$:

$$c^2.t'^2 - r'^2 = \sum_{j=1}^{N} c^2.k^2.\left(t_j - \frac{v}{c^2}.x_j\right)^2 - k^2.\sum_{j=1}^{N}\left(x_j - v.t_j\right)^2 = k^2.\sum_{j=1}^{N}\left[\left(c.t_j - \frac{v}{c}.x_j\right)^2 - \left(x_j - v.t_j\right)^2\right] =$$

$$= k^2.\sum_{j=1}^{N}\left[\left(c^2.t_j^2 - 2.v.t_j.x_j + \frac{v^2}{c^2}.x_j^2\right) - \left(x_j^2 - 2.v.t_j.x_j + v^2.t_j^2\right)\right] = k^2.\sum_{j=1}^{N}\left[c^2.t_j^2 + \frac{v^2}{c^2}.x_j^2 - x_j^2 - v^2.t_j^2\right]$$

$$c^2.t'^2 - r'^2 == k^2.\sum_{j=1}^{N}\left[c^2.t_j^2 + \frac{v^2}{c^2}.x_j^2 - x_j^2 - v^2.t_j^2\right] = k^2.\sum_{j=1}^{N}\left[c^2.t_j^2 - v^2.t_j^2 + \frac{v^2}{c^2}.x_j^2 - x_j^2\right] =$$

$$= k^2.\left(1-\frac{v^2}{c^2}\right).\sum_{j=1}^{N}\left(c^2.t_j^2 - x_j^2\right) = \sum_{j=1}^{N}\left(c^2.t_j^2 - x_j^2\right) = \sum_{j=1}^{N}\left(c^2.t_j^2\right) - \sum_{j=1}^{N}\left(x_j^2\right) = c^2.\sum_{j=1}^{N} t_j^2 - \sum_{j=1}^{N} x_j^2$$

The last expression is precisely the expression that we was looking for: $c^2.t^2 - r^2$.

This procedure demonstrates that vector character of time is not the result of any hypothesis; it comes directly from observing vector properties clearly present inside transformations relating measurements of both inertial observers. It can also be observed, from an epistemological point of view, that time as a spatial vector forms its direction by taking it from the vector velocity v of the moving system O', leaving such parameter with a scalar character and functioning as part of a scaling factor. This behavior of the velocity can be understood due to both observers are on the same inclined line, which would imply this scalar character of v, because for them its value only can be positive or negative.

Another epistemological characteristic of vector time is its dependence on spatial coordinates x, y and z, which means that **it is not an additional independent vector** to our known three-dimensional-spatial universe, a characteristic that appears remarkable because it differs from the Minkowski's four dimensions universe (time as a fourth independent dimension) introduced by Einstein in his special and general theories of relativity. This would mean that if this analysis is correct we are allowed to continue working within our familiar three spatial dimensions in this study for obtaining exact results, and that magnitudes can continue being defined as in Vectorial Physics, but under a relativistic point of view. By observing obtained **results** we would arrive at the following concept of time: Time is forced to behave as a vector with spatial components in each coordinate, when it appears inside an analysis without assumptions which we are going to name now on, Vectorial Lorentz Transformations (VLT), but it can appear also behaving as a scalar value when it is not an element of a transformation such as VLT, in the way we always have known it: as a sequential meter of events. From another point of view, time can also be considered as a vector in the natural way it was referred to by Hongbao Ma. In accordance with his idea and it perfectly applies to our work, Hongbao Ma says: "*this three dimensional time concept is obtained from the mathematical* *conception rather than the ontological existence. Mathematical results are at the epistemological level*" [4]. It is worth to mention that a similar and rigorous presentation of time as a vector, very close to the way we have presented here can be seen in the work done by Bernard Guy [5], published in 2001.

V CORRECT DERIVATION OF LORENTZ TRANSFORMATIONS (2)

By considering time as having the properties of a vector with components on the spatial coordinates, when reflected within the relation between inertial observers with different movements, let's formally obtain the vectorial version for the Lorentz transformations (VLT). So, now we will refer in general to the three-dimensional case, or it could be thought in an n-dimensional case, where system O' moves on a general inclined line and each observer measures his light pulse radio-vector, $\mathbf{r}, \mathbf{r}'$, and his vectorial time, $\mathbf{t}, \mathbf{t}'$. Vectors from now on will be written as boldface letters. The relationships in VLT, previously seen, are easily obtained from **Fig. 3b:**

$$\mathbf{r} = c.\mathbf{t} \qquad \mathbf{r}' = c.\mathbf{t}' \qquad \mathbf{t}' = \frac{\mathbf{r}'}{c} \qquad \mathbf{t} = \frac{\mathbf{r}}{c}$$

$$\left\{\begin{matrix} \mathbf{r}' = k(\mathbf{r} - v.\mathbf{t}) & \Rightarrow & c.\mathbf{t}' = k.\mathbf{t}.(c - v) \\ \mathbf{r} = k(\mathbf{r}' + v.\mathbf{t}') & \Rightarrow & c.\mathbf{t} = k.\mathbf{t}'.(c + v) \end{matrix}\right\} \Rightarrow c^2.\mathbf{t} \bullet \mathbf{t}' = k^2.\mathbf{t}' \bullet \mathbf{t}.(c^2 - v^2) \Rightarrow k^2 = \frac{1}{1 - \frac{v^2}{c^2}}$$

$$\mathbf{r}' = \frac{\mathbf{r} - v.\mathbf{t}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow c.\mathbf{t}' = \frac{c.\mathbf{t} - v.\frac{\mathbf{r}}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \mathbf{t}' = \frac{\mathbf{t} - \frac{v}{c^2}.\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \mathbf{u}' = \frac{d\mathbf{r}'}{dt'} = \frac{d\mathbf{r} - v.d\mathbf{t}}{\left| d\mathbf{t} - \frac{v}{c^2}.d\mathbf{r} \right|} \tag{33}$$

We have previously shown that the following equality also holds as invariant for VLT: $c^2.t'^2 - r'^2 = c^2.t^2 - r^2$. This means that cinematic VLT, composed by expressions, $\mathbf{r}'$, $\mathbf{t}'$ and $\mathbf{u}'$ in (33), are generally valid for a light pulse traveling at c or for any projectile moving at any speed less than c. As a check, the Vectorial Jacobian matrix for any set of components, becomes symmetric and equal to unity, i.e., Letting x^i, be the components measured by O, and x^j with bar, $\bar{x}^j$, be those measured by O', for $i, j = 1, 2, 3$, and for $k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$, we have:

$$\left\{\begin{matrix} \mathbf{r}' = k.(\mathbf{r} - v.\mathbf{t}) & \mathbf{r} = k.(\mathbf{r}' + v.\mathbf{t}') \\ \mathbf{t}' = k.\left(\mathbf{t} - \frac{v}{c^2}.\mathbf{r}\right) & \mathbf{t} = k.\left(\mathbf{t}' + \frac{v}{c^2}.\mathbf{r}'\right) \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} \left(\frac{\partial x^i}{\partial \bar{x}^j}\right) = \begin{vmatrix} k & -k.v \\ -k.\frac{v}{c^2} & k \end{vmatrix} \\ \left(\frac{\partial \bar{x}^j}{\partial x^i}\right) = \begin{vmatrix} k & +k.v \\ +k.\frac{v}{c^2} & k \end{vmatrix} \end{matrix}\right\} \left(\frac{\partial x^i}{\partial \bar{x}^j}\right) = \left(\frac{\partial \bar{x}^j}{\partial x^i}\right) = \frac{1 - \frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}} = 1$$

A final remark on the procedure previously presented: It was not done any assumption. Thus, because these are vectorial relationships, they are generally valid. It is opportune to emphasize the consistency of VLT with Maxwell Equations, which is also demonstrated in Annex A, at the end of this section.

Let's construct the general expressions of components for VLT in three dimensions using spherical coordinates (**Fig. 3b**) by taking as reference the relationships already obtained appearing in (28) and (29), where β is the angle between the inclined trajectory of O' and the plane XY; and α the angle formed by the projection of the inclined trajectory of O' on the plane XY, with the X-axis. When moving origin O' and fixed one O coincide, the light pulse is sent towards the space with generic components x, y, z, The general VLT of the vector time and that of the radio-vector of the pulse of light (or projectile), in three dimensions, become :

$$\begin{aligned} x' &= \frac{x - v.t.\cos\alpha.\cos\beta}{\sqrt{1-\frac{v^2}{c^2}}} \\ y' &= \frac{y - v.t.\sin\alpha.\cos\beta}{\sqrt{1-\frac{v^2}{c^2}}} \\ z' &= \frac{z - v.t.\sin\beta}{\sqrt{1-\frac{v^2}{c^2}}} \end{aligned} \quad \Rightarrow \quad \begin{aligned} t_x &= t.\cos\beta.\cos\alpha \\ t_y &= t.\cos\beta.\sin\alpha \\ t_z &= t\sin\beta \end{aligned} \quad \Rightarrow \quad \begin{aligned} x' &= \frac{x - v.t_x}{\sqrt{1-\frac{v^2}{c^2}}} \\ y' &= \frac{y - v.t_y}{\sqrt{1-\frac{v^2}{c^2}}} \\ z' &= \frac{z - v.t_z}{\sqrt{1-\frac{v^2}{c^2}}} \end{aligned} \tag{34}$$

$$t' = \left| \frac{\mathbf{t} - \frac{v}{c^2}.\mathbf{r}}{\sqrt{1-\frac{v^2}{c^2}}} \right| = \sqrt{\frac{(t_x - \frac{v}{c^2}.x)^2 + (t_y - \frac{v}{c^2}.y)^2 + (t_z - \frac{v}{c^2}.z)^2}{1-\frac{v^2}{c^2}}} \tag{35}$$

The general expressions for **velocities** of the pulse of light or any projectile are obtained from the previous ones:

$$\begin{aligned} u_x' &= \frac{u_x - v.\cos\alpha.\cos\beta}{\sqrt{\left(\cos\alpha.\cos\beta - \frac{v.u_x}{c^2}\right)^2 + \left(\sin\alpha.\cos\beta - \frac{v.u_y}{c^2}\right)^2 + \left(\sin\beta - \frac{v.u_z}{c^2}\right)^2}} \\ u_y' &= \frac{u_y - v.\sin\alpha.\cos\beta}{\sqrt{\left(\cos\alpha.\cos\beta - \frac{v.u_x}{c^2}\right)^2 + \left(\sin\alpha.\cos\beta - \frac{v.u_y}{c^2}\right)^2 + \left(\sin\beta - \frac{v.u_z}{c^2}\right)^2}} \end{aligned} \tag{36}$$

$$u'_z = \frac{u_z - v.\sin\beta}{\sqrt{\left(\cos\alpha.\cos\beta - \frac{v.u_x}{c^2}\right)^2 + \left(\sin\alpha.\cos\beta - \frac{v.u_y}{c^2}\right)^2 + \left(\sin\beta - \frac{v.u_z}{c^2}\right)^2}}$$

Let's particularize these general results to the conditions from where the original LT were obtained **(Fig. 3a)**. If we re-establish such conditions (the system O' moving along the X axis, and the light pulse sent to space), i.e., for $\alpha=\beta=0$, we will obtain the VLT version of the original Lorentz transformations:

$$x' = \frac{x - v.t}{\sqrt{1-\frac{v^2}{c^2}}};\quad y' = \frac{y}{\sqrt{1-\frac{v^2}{c^2}}};\quad z' = \frac{z}{\sqrt{1-\frac{v^2}{c^2}}}\qquad \begin{matrix} t_x = t \\ t_y = 0 \\ t_z = 0 \end{matrix}\qquad t' = \sqrt{\frac{(t_x - \frac{v}{c^2}.x)^2 + (\frac{v}{c^2}.y)^2 + (\frac{v}{c^2}.z)^2}{1-\frac{v^2}{c^2}}} \tag{37}$$

$$u'_x = \frac{u_x - v}{\sqrt{\left(1-\frac{v.u_x}{c^2}\right)^2 + \left(\frac{v.u_y}{c^2}\right)^2 + \left(\frac{v.u_z}{c^2}\right)^2}}\qquad u'_y = \frac{u_y}{\sqrt{\left(1-\frac{v.u_x}{c^2}\right)^2 + \left(\frac{v.u_y}{c^2}\right)^2 + \left(\frac{v.u_z}{c^2}\right)^2}}$$

$$u'_z = \frac{u_z}{\sqrt{\left(1-\frac{v.u_x}{c^2}\right)^2 + \left(\frac{v.u_y}{c^2}\right)^2 + \left(\frac{v.u_z}{c^2}\right)^2}}\qquad u'^2_x + u'^2_y + u'^2_z = u_x^2 + u_y^2 + u_z^2 = c^2 \tag{38}$$

Let's check the last relationship in (38), which is valid only for photons. In such equation is then implied that the velocity of light measured by any of the two observers should be the same, c (In general for any other projectile, $u'^2 \neq u^2$), In fact, on the basis of which O measures, $u_x^2 + u_y^2 + u_z^2 = c^2$, then O' will measure:

$$u'^2_x + u'^2_y + u'^2_z = \frac{(u_x - v)^2 + u_y^2 + u_z^2}{\left(1-\frac{v.u_x}{c^2}\right)^2 + \left(\frac{v.u_y}{c^2}\right)^2 + \left(\frac{v.u_z}{c^2}\right)^2} = \frac{u_x^2 - 2.v.u_x + v^2 + u_y^2 + u_z^2}{1 - \frac{2.v.u_x}{c^2} + \frac{v^2.u_x^2}{c^4} + \frac{v^2.u_y^2}{c^4} + \frac{v^2.u_z^2}{c^4}} =$$

$$= \frac{\left(u_x^2 + u_y^2 + u_z^2\right) - 2.v.u_x + v^2}{1 - \frac{2.v.u_x}{c^2} + \frac{v^2}{c^4}.\left(u_x^2 + u_x^2 + u_x^2\right)} = \frac{c^2 - 2.v.u_x + v^2}{1 - \frac{2.v.u_x}{c^2} + \frac{v^2}{c^2}} = \frac{c^2.\left(1 - \frac{2.v.u_x}{c^2} + \frac{v^2}{c^2}\right)}{1 - \frac{2.v.u_x}{c^2} + \frac{v^2}{c^2}} = c^2$$

And by using the particularized equations in (31) for the LT , the demonstration of the space-time interval invariance readily follows:

$$r'^2 - c^2.t'^2 = \frac{(x-v.t)^2 + y^2 + z^2}{1-\frac{v^2}{c^2}} - c^2.\frac{(t-\frac{v}{c^2}.x)^2 + (\frac{v}{c^2}.y)^2 + (\frac{v}{c^2}.z)^2}{1-\frac{v^2}{c^2}} =$$

$$r'^2 - c^2.t'^2 = \frac{x^2+y^2+z^2-2.v.t.x+v^2.t^2-c^2.t^2+2.v.t.x-\frac{v^2}{c^2}.(x^2+y^2+z^2)}{1-\frac{v^2}{c^2}} =$$

Reordering and simplifying, we have

$$r'^2 - c^2.t'^2 = \frac{(x^2+y^2+z^2)\left[1-\frac{v^2}{c^2}\right] - \left[1-\frac{v^2}{c^2}\right]c^2.t^2}{1-\frac{v^2}{c^2}} = r^2 - c^2.t^2$$

When comparing equations (31) and (32), with the original LT equations, repeated in (33) and (34):

$$x' = \frac{x-v.t}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \boxed{y' = y \qquad z' = z} \qquad t' = \frac{t-\frac{v.x}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = t.\frac{1-\frac{v}{c^2}.u_x}{\sqrt{1-\frac{v^2}{c^2}}} \tag{39}$$

$$u'_x = \frac{u_x - v}{1-\frac{v.u_x}{c^2}} \qquad \boxed{u'_y = \frac{u_y.\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v.u_x}{c^2}} \qquad u'_z = \frac{u_z.\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v.u_x}{c^2}}} \tag{40}$$

The first thing we realize in equations (37) is that components y, z, measured by the fixed observer are different to those of y', z', measured by the moving observer, thus, revealing that original LT's "statements" (assumptions) are wrong and their consequences. We can observe that the expression of time in (37) is completely different to that of Lorentz in (40). And of course, the obtained expressions, according to this work, for velocity components u'_x, u'_y, u'_z in equations (38), are also different of those presented for LT in (40).

VI. CONCLUSION

1) Given that our procedure to obtain the modified Lorentz Transformations, named by us as Vectorial Lorentz Transformations (VLT) does not assume that y and z coordinates transform in a Galilean manner, it sufficiently demonstrates that in the classical derivation of Lorentz Transformations (LT) they were needless and wrong. Therefore, classical LT is

reduced to be only valid for a one-dimensional space, where assumptions are not needed. So, its validity must not be extrapolated to a general configuration. This means that Lorentz Transformations as we have known until now are erroneous when applied to our three-dimensional space, and in general, they are erroneous when applied to spaces with more than one dimension.

2) If we consider the Lorentz Transformations as a central part of the Special Theory of Relativity (STR), as it was considered by Einstein in [3], then if this work is accepted as an exact one, STR should be also corrected as it was LT in this work.

3) But, given that time has indeed vector properties, and in this work we have demonstrated that they are those of a vector depending on the spatial coordinates. This means that time, as a vector, is not perpendicular to the spatial coordinates, or shortly, it is not an independent vector. Therefore, the consideration that time constitutes an additional dimension to our three dimensions universe should be considered a wrong statement.

4) The fact that time does not behave as an independent vector (it is not perpendicular to spatial axes) not only would imply that Einstein's Special Theory of Relativity (based in the four-dimensional Minkowski space) is not a correct theory in Physics but also, because of this same reason, should be it the Einstein's General Theory of Relativity (GTR).

Comment: It is conceivable that when in 1905 Einstein established his remarkable concept of the variation of mass with its velocity [1], whose expression it has been demonstrated to be also wrong [3] he was actually looking for the one-to-one variation of physical magnitudes between classic and relativistic physics through Lorentz factors. By the way, In this same issue there is a work that corrects Lorentz factors used in SRT. In an author's speculative opinion, as Einstein discarded longitudinal definition of relativistic mass (erroneously [3]) without any explanation, he seems later to have abandoned SRT due to some observed inconsistencies and limitations inside LT, and might be this was one of the reasons he had for developing the General Theory of Relativity (GRT) trying to avoid such type of limitations in his research.

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ANNEX A

CONSISTENCY OF VECTORIAL LORENTZ TRANSFORMATIONS AND INCONSISTENCY OF LORENTZ TRANSFORMATIONS

Given that we are working with vectors, it is suitable to obtain a Wave Equation presentation in function of the light pulse radio-vector and time vector. Thus,

$$\mathbf{r} = x.\mathbf{i} + y.\mathbf{j} + z.\mathbf{k} \quad \Rightarrow \quad r = \sqrt{x^2 + y^2 + z^2} \quad \Rightarrow \quad \frac{\partial r}{\partial x} = \frac{x}{r}; \; \frac{\partial r}{\partial y} = \frac{y}{r}; \; \frac{\partial r}{\partial z} = \frac{z}{r};$$

In this way, each one of the components of operator ∇ can be represented as depending on both vectors. Let's work to arrive at equations depending only on r and t. For this, we will develop the expressions of the components x, y, z:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial}{\partial t}\frac{\partial t}{\partial x}; \; \frac{\partial}{\partial y} = \frac{\partial}{\partial r}\frac{\partial r}{\partial y} + \frac{\partial}{\partial t}\frac{\partial t}{\partial y}; \; \frac{\partial}{\partial z} = \frac{\partial}{\partial r}\frac{\partial r}{\partial z} + \frac{\partial}{\partial t}\frac{\partial t}{\partial z}; \text{ Where, } \frac{\partial t}{\partial x} = \frac{\partial t}{\partial y} = \frac{\partial t}{\partial z} = 0$$

So, the operator $\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$ can be expressed as:

$$\nabla = \frac{\partial}{\partial r}\frac{\partial r}{\partial x}\mathbf{i} + \frac{\partial}{\partial r}\frac{\partial r}{\partial y}\mathbf{j} + \frac{\partial}{\partial r}\frac{\partial r}{\partial z}\mathbf{k} = \frac{\partial}{\partial r}\frac{x}{r}\mathbf{i} + \frac{\partial}{\partial r}\frac{y}{r}\mathbf{j} + \frac{\partial}{\partial r}\frac{z}{r}\mathbf{k} = \frac{\partial}{\partial r}\frac{\mathbf{r}}{r} \quad \Rightarrow \quad \nabla \bullet \nabla = \nabla^2 = \frac{\partial^2}{\partial r^2}$$

Thus, Wave Equation can be put in a simpler manner, only as function of r and t:

$$\frac{\partial^2 \varepsilon}{\partial r^2} - \frac{1}{c^2}.\frac{\partial^2 \varepsilon}{\partial t^2} = 0; \quad \text{For:} \quad \left\{ \begin{array}{lll} \mathbf{r}' = \dfrac{\mathbf{r} - v.\mathbf{t}}{\sqrt{1 - \frac{v^2}{c^2}}} \;\Rightarrow\; \dfrac{\partial r'}{\partial t} = \dfrac{-v}{\sqrt{1 - \frac{v^2}{c^2}}} & & \dfrac{\partial r'}{\partial r} = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \\ \mathbf{t}' = \dfrac{\mathbf{t} - \frac{v}{c^2}.\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} \;\Rightarrow\; \dfrac{\partial t'}{\partial t} = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}; & & \dfrac{\partial t'}{\partial r} = \dfrac{-\frac{v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}; \end{array} \right\}$$

By applying Chain rule for partial derivation, respect to variables r, t:

$$\frac{\partial \varepsilon}{\partial r} = \frac{\partial \varepsilon}{\partial r'}\frac{\partial r'}{\partial r} + \frac{\partial \varepsilon}{\partial t'}\frac{\partial t'}{\partial r} \qquad \frac{\partial \varepsilon}{\partial t} = \frac{\partial \varepsilon}{\partial r'}\frac{\partial r'}{\partial t} + \frac{\partial \varepsilon}{\partial t'}\frac{\partial t'}{\partial t}$$

Substituting values previously obtained:

$$\frac{\partial \varepsilon}{\partial r} = \frac{\partial \varepsilon}{\partial r'} \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{\partial \varepsilon}{\partial t'} \frac{-\frac{v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}; \qquad \frac{\partial \varepsilon}{\partial t} = \frac{\partial \varepsilon}{\partial r'} \frac{-v}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{\partial \varepsilon}{\partial t'} \frac{1}{\sqrt{1-\frac{v^2}{c^2}}};$$

By differentiating again, in order to form all required quadratics components of Wave Equation:

$$\frac{\partial^2 \varepsilon}{\partial r^2} = \frac{1}{\left(1-\frac{v^2}{c^2}\right)} \left(\frac{\partial^2 \varepsilon}{\partial r'^2} + \frac{v^2}{c^4} \frac{\partial^2 \varepsilon}{\partial t'^2} - 2\frac{v}{c^2} \frac{\partial^2 \varepsilon}{\partial r' \, \partial t'} \right)$$

$$\frac{\partial^2 \varepsilon}{\partial t^2} = \frac{1}{\left(1-\frac{v^2}{c^2}\right)} \left(v^2 \frac{\partial^2 \varepsilon}{\partial r'^2} + \frac{\partial^2 \varepsilon}{\partial t'^2} - 2.v.\frac{\partial^2 \varepsilon}{\partial r' \, \partial t'} \right)$$

And substituting these obtained expressions in Wave Equation, we finally have:

$$\frac{\partial^2 \varepsilon}{\partial r^2} - \frac{1}{c^2}.\frac{\partial^2 \varepsilon}{\partial t^2} =$$

$$= \frac{1}{\left(1-\frac{v^2}{c^2}\right)} \left(\frac{\partial^2 \varepsilon}{\partial r'^2} + \frac{v^2}{c^4} \frac{\partial^2 \varepsilon}{\partial t'^2} - 2\frac{v}{c^2} \frac{\partial^2 \varepsilon}{\partial r' \, \partial t'} \right) - \frac{1}{c^2}.\frac{1}{\left(1-\frac{v^2}{c^2}\right)} \left(v^2 \frac{\partial^2 \varepsilon}{\partial r'^2} + \frac{\partial^2 \varepsilon}{\partial t'^2} - 2.v.\frac{\partial^2 \varepsilon}{\partial r' \, \partial t'} \right)$$

$$\boxed{\frac{\partial^2 \varepsilon}{\partial r^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon}{\partial t^2} = \frac{\partial^2 \varepsilon}{\partial r'^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon}{\partial t'^2}}$$

In this way, it is shown that VLT are consistent with Wave Equation, which come from Maxwell Equations. So, it has been shown that Wave Equation under VLT has the same presentation for one and another observer, independent of the path of the moving observer and also independent of the light pulse direction. This means that VLT meet Einstein relativistic postulates and are compatible with Maxwell Equations.

The problem that LT have, according to our development, is precisely the following assumptions: $y' = y$ and $z' = z$ that originates $\frac{\partial^2 \varepsilon_x}{\partial y^2} = \frac{\partial^2 \varepsilon_x}{\partial y'^2}$ and $\frac{\partial^2 \varepsilon_x}{\partial z^2} = \frac{\partial^2 \varepsilon_x}{\partial z'^2}$. They don't allow a vectorial treatment through variables r, t. For instance in two dimensions, according to Lorentz, the displacement of light measured by fixed observer at O, is $\mathbf{r} = x.\mathbf{i} + y.\mathbf{j}$, and the corresponding measurement done by moving one at O' is:

$$\mathbf{r}' = x'.\mathbf{i} + y'.\mathbf{j} = \frac{x - v.t}{\sqrt{1-\frac{v^2}{c^2}}}\mathbf{i} + y.\mathbf{j} = \frac{x}{\sqrt{1-\frac{v^2}{c^2}}}\mathbf{i} + y.\mathbf{j} - \frac{-v.t}{\sqrt{1-\frac{v^2}{c^2}}}\mathbf{i}.$$

By observing carefully this last equation, we conclude that it is not possible to obtain an explicit expression of $\mathbf{r}'$ as function of $\mathbf{r}$ and $\mathbf{t}$. Thus, it can not be possible to obtain an expression for $\frac{\partial r'}{\partial r}$, neither for $\frac{\partial t'}{\partial r}$. In these circumstances we can not continue with the procedure of constructing the vectorial version of the original LT. It can be shown that LT really are not invariant to the Wave equation. Although in some books appears a "demonstration" of the consistency of the LT with Wave Equation, this is not quite general, this is actually a demonstration that is valid only for the particular case of one dimension: the X axis, in where assumptions are not necessary. The chosen example for such demonstration is always presented without any variation: An observer located at the origin O' of the moving system, which moves on the X axis, and a light pulse is sent to the "space" parallel to the X axis when he coincides with fixed origin O. For instance, if this presentation is changed, for example, by establishing that the pulse of light is going parallel to the Z axis, maintaining the moving observer on the X axis, the "demonstration" fails. For showing this, we will work out a known example taken from basic electromagnetic theory:

A) Let an electromagnetic plane wave move on Z axis at light speed, $z = ct$, such that the electric field on Y axis, $\varepsilon_y = \varepsilon_o.\sin k.(z - c.t)$, depends only on the Z coordinate and time. So, field characteristics will be: $\varepsilon_x = 0$; $\varepsilon_y = \varepsilon_y(z,t)$; $\varepsilon_z = 0$; $x = y = 0$. Suppose that the system O' is moving along the X axis at a velocity v and let's assume, $z' = z$, in order to be under the same premises of LT.

The relationships that hold for this case, according to LT, are:

$$x' = \frac{-v.t}{\sqrt{1-\frac{v^2}{c^2}}};\ x = 0;\ y' = y = 0;\ z' = z;\ t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \frac{\partial x'}{\partial t} = \frac{-v}{\sqrt{1-\frac{v^2}{c^2}}};\ \frac{\partial t'}{dt} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}};\ \frac{\partial t'}{\partial x} = 0$$

With these premises, we can write: $\frac{\partial \varepsilon_y}{\partial x} = \frac{\partial \varepsilon_y}{\partial y} = 0$, and similarly, $\frac{\partial y'}{\partial t} = 0$; $\frac{\partial t'}{\partial z} = 0$; Given that time is not an explicit variable in the expression of z', then $\frac{\partial z'}{\partial t} = 0$; and because $z' = z$, then: $\frac{\partial z'}{\partial z} = 1$. In this way, all the equations corresponding to Wave Equation in function of the coordinate components are reduced to:

$$\frac{\partial^2 \varepsilon_y}{\partial x^2} + \frac{\partial^2 \varepsilon_y}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \varepsilon_y}{\partial t^2} = 0 \qquad \Rightarrow \qquad \frac{\partial^2 \varepsilon_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 \varepsilon_y}{\partial t^2} = 0$$

Let's try to build the Wave Equation under prime variables. By using the Chain rule:

$$\frac{\partial \varepsilon_y}{\partial z}=\frac{\partial \varepsilon_y}{\partial x'}\frac{\partial x'}{\partial z}+\frac{\partial \varepsilon_y}{\partial y'}\frac{\partial y'}{\partial z}+\frac{\partial \varepsilon_y}{\partial z'}\frac{\partial z'}{\partial z}+\frac{\partial \varepsilon_y}{\partial t'}\frac{\partial t'}{\partial z}=\frac{\partial \varepsilon_y}{\partial z'}\frac{\partial z'}{\partial z}=\frac{\partial \varepsilon_y}{\partial z'} \quad \Rightarrow \quad \frac{\partial^2 \varepsilon_y}{\partial z^2}=\frac{\partial^2 \varepsilon_y}{\partial z'^2}$$

$$\frac{\partial \varepsilon_y}{\partial t}=\frac{\partial \varepsilon_y}{\partial x'}\frac{\partial x'}{\partial t}+\frac{\partial \varepsilon_y}{\partial y'}\frac{\partial y'}{\partial t}+\frac{\partial \varepsilon_y}{\partial z'}\frac{\partial z'}{\partial t}+\frac{\partial \varepsilon_y}{\partial t'}\frac{\partial t'}{\partial t}=\frac{\partial \varepsilon_y}{\partial x'}\frac{\partial x'}{\partial t}+\frac{\partial \varepsilon_y}{\partial t'}\frac{\partial t'}{\partial t}$$

$$\frac{\partial^2 \varepsilon_y}{\partial t^2}=\frac{\partial^2 \varepsilon_y}{\partial x'^2}\frac{\partial x'^2}{\partial t^2}+\frac{\partial^2 \varepsilon_y}{\partial t'}\frac{\partial t'^2}{\partial t^2}-2.\left[\frac{\partial^2 \varepsilon_y}{\partial x'\partial t'}\frac{\partial x'}{\partial t}\frac{\partial t'}{\partial t}\right]$$

Substituting by their values:

$$\frac{\partial^2 \varepsilon_y}{\partial t^2}=\frac{1}{\left(1-\frac{v^2}{c^2}\right)}\left(v^2\frac{\partial^2 \varepsilon_y}{\partial x'^2}+\frac{\partial^2 \varepsilon_y}{\partial t'^2}-2.v.\frac{\partial^2 \varepsilon_y}{\partial x'\,\partial t'}\right),$$ and Introducing these results, a different presentation of the Wave Equation for the prime variables is obtained: contrary to what is expected:

$$\frac{\partial^2 \varepsilon_y}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 \varepsilon_y}{\partial t^2}=\frac{\partial^2 \varepsilon_y}{\partial z'^2}-\frac{1}{c^2}\frac{1}{\left(1-\frac{v^2}{c^2}\right)}\left(v^2\frac{\partial^2 \varepsilon_y}{\partial x'^2}+\frac{\partial^2 \varepsilon_y}{\partial t'^2}-2.v.\frac{\partial^2 \varepsilon_y}{\partial x'\,\partial t'}\right)$$

This result shows how the original LT **are not really consistent** with the Maxwell Equations, because it does not preserve the structure of Wave Equation. The reason for LT having this failure is clear: the assumption $z'=z$ is wrong.

B) Let's do the same job but through the VLT. Expressing the movement of O' and that of the light pulse in a vectorial form, we get:

$\alpha=\beta=x=y=t_y=t_z=0; \Rightarrow \mathbf{t}=t.\mathbf{i}; \quad \mathbf{r}=z.\mathbf{k} \Rightarrow z=r$; By applying: $\left\{\begin{matrix}\mathbf{t}'=k.(\mathbf{t}-\frac{v}{c^2}.\mathbf{r})\\ \mathbf{r}'=k.(\mathbf{r}-v.\mathbf{t})\end{matrix}\right\}$

$$\mathbf{t}'=\frac{t.\mathbf{i}-\frac{v}{c^2}.r.\mathbf{k}}{\sqrt{1-\frac{v^2}{c^2}}};\quad \mathbf{r}'=\frac{r.\mathbf{k}-v.t.\mathbf{i}}{\sqrt{1-\frac{v^2}{c^2}}} \quad \Rightarrow \quad \left\{\begin{matrix}\frac{\partial t'}{\partial t}=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}; & \frac{\partial t'}{\partial r}=\frac{-\frac{v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}\\ \frac{\partial r'}{\partial r}=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}; & \frac{\partial r'}{\partial t}=\frac{-v}{\sqrt{1-\frac{v^2}{c^2}}}\end{matrix}\right\}$$

Wave Equation, in function of r,t had become: $\frac{\partial^2 \varepsilon_y}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2 \varepsilon_y}{\partial t^2} = 0$. Operating as before, and substituting values, primed Wave Equation is consistently obtained:

$$\frac{\partial^2 \varepsilon}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2 \varepsilon}{\partial t^2} = \left[\frac{\partial^2 \varepsilon}{\partial r'^2}\frac{\partial r'^2}{\partial r^2} + \frac{\partial^2 \varepsilon}{\partial t'^2}\frac{\partial t'^2}{\partial r^2} - 2.\frac{\partial^2 \varepsilon}{\partial r'.\partial t'}\frac{\partial r'.\partial t'}{\partial r^2}\right] - \frac{1}{c^2}\left[\frac{\partial^2 \varepsilon}{\partial r'^2}\frac{\partial r'^2}{\partial t^2} + \frac{\partial^2 \varepsilon}{\partial t'^2}\frac{\partial t'^2}{\partial t^2} - 2.\frac{\partial^2 \varepsilon}{\partial r'.\partial t'}\frac{\partial r'.\partial t'}{\partial t^2}\right]$$

$$\frac{\partial^2 \varepsilon}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2 \varepsilon}{\partial t^2} = \frac{\partial^2 \varepsilon}{\partial r'^2} - \frac{1}{c^2}\frac{\partial^2 \varepsilon}{\partial t'^2}$$

As so it was expected. This also means that VLT are truly consistent with Wave Equation and in general with Maxwell Equations, and not thus LT are.