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Letter

Superposition and Accumulation

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ABSTRACT: There are two kinds of addition, namely, superposition and accumulation. Superposition is the kind of addition that the addendums are simultaneous and at the same point and have the same property. Accumulation is the kind of addition that the addendums are not simultaneous or not at the same point. The commutative law holds for superposition without exception, but it does not necessarily for accumulation. In classic mechanics, the addendums of superposition are all about a single particle, the addition whose addendums are about different particles is not superposition unexceptionally; while the addendums of accumulation are not necessarily about a single particle. The article is very important in understanding and using the superposition principle.

KEYWORDS: superposition; accumulation; addition; commutative law; superposition principle.

I THE NECESSARY CONDITIONS OF SUPERPOSITION

I.1. The superposition principle *[1]* ***of motion.*** This can be stated as follows: the total motion of a particle is the superposition of its independent motions ***simultaneously***, motions at different times or at different points can never superpose. Let me explain it in examples.

Example 1: the projectile motion. If we throw a particle with an initial velocity $\mathbf{v}_0$, it will fly a definite time and distance (assuming gravitational acceleration $\mathbf{g}$ is a constant vector and resistant forces omitted). Using Newton's second law of motion, we get:

$$\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt}, \mathbf{G} = m\mathbf{g} = m\frac{d\mathbf{v}}{dt} \Rightarrow d\mathbf{v} = \mathbf{g}dt$$
$$\Rightarrow \int d\mathbf{v} = \int \mathbf{g}dt \Rightarrow \int_{\mathbf{v}_0}^{\mathbf{v}} d\mathbf{v} = \int_0^t \mathbf{g}dt \Rightarrow \mathbf{v} = \mathbf{v}_0 + \mathbf{g}t = \frac{d\mathbf{r}}{dt}$$
$$\Rightarrow \mathbf{r} - \mathbf{r}_0 = \mathbf{v}_0 t + \frac{1}{2}\mathbf{g}t^2$$

The total motion of the projectile is the superposition of its two independent motions ***simultaneously***, i.e. the uniform rectilinear motion with velocity $\mathbf{v}_0$ and the free-fall motion with constant acceleration $\mathbf{g}$.

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Example 2: the fixed point rotation of a rigid body. The fixed point rotation of a rigid body can be considered as the superposition of three independent revolutions ***simultaneously***, namely, the precession, the nutation, and the rotation.

Example 3: the principle of wave superposition. If two or more traveling waves are moving through a medium and combine at a given point, the resultant displacement of the medium at that point is the sum of the displacements of the individual waves. Apparently, the principle of wave superposition is a special case of the principle of motion superposition.

I.2. The superposition principle of force: the total force acting on a particle due to a group of particles equals the sum of the individual forces due to all the particles in the group.

I.3 The superposition principle of electric intensity: the total electric intensity at a point due to multiple point charges equals the sum of the electric intensities at the point due to all the individual point charges.

I.4 The superposition principle of electric potential: the total electric potential at a point due to multiple point charges equals the sum of the electric potentials at the point due to all the individual point charges.

I.5 The superposition principle of magnetic induction intensity: the total magnetic induction intensity at a point due to multiple moving point charges equals the sum of the magnetic induction intensities at the point due to all the individual moving point charges.

Based on all the above superposition principles, we conclude that the necessary conditions of superposition are:

1) *Simultaneity*---all the addendums in superposition exist simultaneously;

2) *At the same point* --- all the addendums in superposition are at the same point;

3) *Having the same property*--- all the addendums in superposition are of the same kind, e.g., force can superimpose only with force.

II THE RELATION BETWEEN SUPERPOSITION AND ACCUMULATION

II.1 Classification. There are two kinds of addition, namely, superposition and accumulation. Superposition is the kind of addition that the addendums are simultaneous and at the same point and have the same property. Accumulation is the kind of addition that the addendums are not simultaneous or not at the same point. Accumulation can be divided into three subdivisions.

1) *Temporal accumulation*. Temporal accumulation is the kind of accumulation whose addendums do not exist simultaneously but are at the same place, e.g., charge $q = \int_0^t idt$, where *i* is current and *t* is time.

2) *Spatial accumulation*. Spatial accumulation is the kind of accumulations whose addendums exist simultaneously but are not at the same point, e.g., the momentum of an *n*-particle system $\mathbf{p}=\sum_{i=1}^{n}\mathbf{p}_i=\sum_{i=1}^{n}m_i\mathbf{v}_i$ and its mass $m=\sum_{i=1}^{n}m_i$, where $\mathbf{r}_i$ is the position vector of the *i*th particle , m_i is the mass of the *i*th particle, $\mathbf{v}_i$ is the velocity of the *i*th particle, $\mathbf{p}_i$ is the momentum of the *i*th particle.

3) *Random accumulation*. Random accumulation is the kind of accumulation whose addendums do not exist simultaneously and are not at the same place, e.g., the displacement of a particle $\Delta\mathbf{r}=\mathbf{r}(t+\Delta t)-\mathbf{r}(t)\ (\Delta t>0)$ where $\mathbf{r}$ is the position vector of the particle and t is time. Subtraction is in essence a kind of addition. When you subtract a quantity, you actually add the negative of the quantity.

II.2 The commutative law holds for superposition unavoidably, but not necessarily for accumulation. It is valid for some accumulations but not valid for others. As an instance, there is an important proposition in theoretical mechanics---The commutative law holds for addition of infinitesimal rotation [2-4], but it does not for addition of finite rotation. That is because addition of infinitesimal rotation is superposition, which meets the commutative law without exception; addition of finite rotation is an example of accumulation which does not meet the commutative law. $\mathbf{p}=\sum_{i=1}^{n}\mathbf{p}_i=\sum_{i=1}^{n}m_i\mathbf{v}_i$ and $m=\sum_{i=1}^{n}m_i$ are two examples of accumulation that meets the commutative law.

II.3 In classic mechanics, the addendums of superposition are about single particle; the addition whose addendums are about different particles is not superposition at all. The addendums of some accumulation are about a single particle, e.g., $\Delta\mathbf{r}=\mathbf{r}(t+\Delta t)-\mathbf{r}(t)\ (\Delta t>0)$; while the addendums of some accumulation are not about a single particle, e.g., $\mathbf{p}=\sum_{i=1}^{n}\mathbf{p}_i=\sum_{i=1}^{n}m_i\mathbf{v}_i$ and $m=\sum_{i=1}^{n}m_i\ (n\geq 2)$.

III CONCLUSION

The necessary conditions of superposition are simultaneity and at the same point and having the same property. Motions at different times or at different points can never superpose. The addendums at different times or at different points can never superpose. Accumulation is the kind of addition that the addendums are not simultaneous or not at the same point.

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