JVR

Journal of Vectorial Relativity
JVR **2** (2007) **3** 57-69

Can a Photon Escape from a Black Hole?

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ABSTRACT: Because of the constancy of its speed it has been shown in previous works that photon has some limitations in its motion. In this new work it is investigated the possible trajectories of a photon in the environs of a Black Hole. The probable allowed trajectories of a photon going towards a Black Hole are analyzed, and similarly, the possible trajectories that photon can follow in its escape from the environs of a black hole are studied. The minimum value of photon's linear momentum for escaping from the gravitational field of a Black Hole was calculated, as well as the maximum angle of inclination that photon in its takeoff of the Black Hole was examined.
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KEYWORDS: Physics, Vectorial Relativity, Gravitation, Schwarzschild radius, Black Hole, Photon.

I INTRODUCTION

In this part we will try to enlarge the study for photon when a very massive body, with mass M attracts it. The massive body is considered fixed and it is assumed that its center coincides with the center of mass of the system of these two bodies.

We had seen in previous work [1] [2], that gravitational field $\mathcal{G}$ exerted by a fixed body of mass M over the mass of a photon going at a speed c makes the latter have a curvilinear path that cuts gravitational field lines produced by mass M. But in turn, gravitational field is modified by the value of photon's linear moment, $p = m.c$: $\mathcal{G} = \dfrac{\frac{2.G.M}{r^2}}{\frac{p}{p_0}+\frac{p_0}{p}}$. It is noteworthy to bear in mind that this comes out from working on the differential equation for curvilinear photon's motion, $\dfrac{d^2r}{dt^2} - r.\omega^2 + \dfrac{\mathcal{G}}{c^2}\omega^2.r^2 = 0$, and from Newton's gravitational force: $F = m.\mathcal{G} = \dfrac{p}{c}.\mathcal{G} = \dfrac{p}{c}.\dfrac{\frac{2.G.M}{r^2}}{\frac{p}{p_0}+\frac{p_0}{p}}$.

We have shown in previous work [3] that ***elliptical*** paths are forbidden for photons in its curvilinear motion around a black hole, mainly because the constant speed of photons imposed limitations to this type of motion. It was obtained there that photon apparently only can have the following type of paths: hyperbolical for $r > 2.r_0$, parabolic for $r \cong 2.r_0$, and spiral for $r < 2.r_0$ until it reaches $r = r_0$. At this moment in the last case, under determined circumstances of inclination it converts its path into circular of constant radius r_0. But, when photon's approximation towards BH is in such a way that it is unavoidable cutting the circle at $r = r_0$, then a route through the region where $r < r_0$ describing a spiral concentric path is only allowed until it strikes the Black Hole surface at $r|_{BH} = r_{BH} < r_0$. This

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reflection completes this particular brief inspection on the curvilinear path of photon. Analysis of photon's approximation to a BH along a ***rectilinear*** path is presented in next section.

II RADIAL APPROXIMATION OF A PHOTON TOWARDS A BLACK HOLE

When photon's movement is aligned with the rectilinear radial gravitational field lines, produced by a massive body, it has a null angular velocity, $\omega = 0$, and it meets the following particular equation derived for photon's motion in [2]:

$$\frac{d^2 r}{dt^2} - r.\omega^2 + \frac{\mathcal{G}}{c^2}\omega^2.r^2 = 0 \;\Rightarrow\; \frac{d^2 r}{dt^2} = 0 \tag{1}$$

This is clearly true because of the constancy of photon's speed. In this case, field lines are not crossed by any photon's path and thus force on it seems not to be influenced by any other cause different of photon's mass variation indirectly present through its linear momentum. In the same order of ideas, we can say that value of the gravitational field depends neither on r_0 nor on p_0, because these particular values don't have any meaning for an attracted photon moving along a rectilinear path, namely, substitution of r_0, p_0 by r, p in expression $\mathcal{G} = \dfrac{\frac{2.G.M}{r^2}}{\frac{p}{p_0} + \frac{p_0}{p}}$, returns the known Newton's expression of gravitational field $\mathcal{G} = \dfrac{G.M}{r^2}$. Thus, for the rectilinear motion of an attracted photon as measurements ratify, field $\mathcal{G}$ is given by Newton's definition. In like manner, attracting force on a photon in rectilinear motion should be given by the Newton's expression: $\mathcal{F} = m.\mathcal{G} = m.\dfrac{G.M}{r^2} = \dfrac{p}{c}.\mathcal{G} = \dfrac{p}{c}.\dfrac{G.M}{r^2}$.

III CURVILINEAR PATH OF A PHOTON GOING TOWARDS A BLACK HOLE

Let the gravitational field $\mathcal{G}$ applied on a moving photon be given by the expression $\mathcal{G} = \dfrac{\frac{2.G.M}{r^2}}{\frac{p}{p_0} + \frac{p_0}{p}}$, with definitions of all variables for curvilinear motion. As in radial case, when reference point is the center of the BH, $r_0 = 0$, **gravitational field** $\mathcal{G}$ in curvilinear motion, **becomes infinite** and similarly, the final value of photon's linear momentum reaches infinite, $p_0 = \infty$. By remembering the expression of the linear momentum in curvilinear motion [2], the following formulas apply:

$$\frac{p}{p_0} = \left[\sqrt{\left(\frac{G.M}{c^2}.\left(\frac{1}{r_0} - \frac{1}{r}\right)\right)^2 + 1} - \frac{G.M}{c^2}.\left(\frac{1}{r_0} - \frac{1}{r}\right)\right]; \quad \frac{p_0}{p} = \left[\sqrt{\left(\frac{G.M}{c^2}.\left(\frac{1}{r_0} - \frac{1}{r}\right)\right)^2 + 1} + \frac{G.M}{c^2}.\left(\frac{1}{r_0} - \frac{1}{r}\right)\right]$$

$$\frac{p_0}{p_\infty}+\frac{p_\infty}{p_0}=2.\sqrt{\left(\frac{G.M}{c^2}.\left(\frac{1}{r_0=0}-\frac{1}{r_\infty}\right)\right)^2+1}=2.\sqrt{\left(\frac{G.M}{c^2}.(\infty)\right)^2+1}=\infty \qquad (2)$$

$$\lim_{r\to 0} \mathcal{G}=\lim_{r\to 0}\left(\frac{\frac{2.G.M}{r^2}}{\frac{p}{p_0}+\frac{p_0}{p}}\right)=\lim_{r\to 0}\frac{\frac{2.G.M}{r^2}}{2.\sqrt{\left(\frac{G.M}{c^2.r}\right)^2+1}}=\frac{\frac{c^2}{r}}{\sqrt{1+\left(\frac{c^2.r}{G.M}\right)^2}}=\infty \qquad (3)$$

We can observe from the results presented in previous sections that the gravitational field, $\mathcal{G}=\frac{G.M}{r^2}\rightarrow\infty^2$, for rectilinear or radial motion of a photon approximating to the center of the BH, goes infinite more rapidly than the gravitational field, $\mathcal{G}=\frac{\frac{2.G.M}{r^2}}{\frac{p}{p_0}+\frac{p_0}{p}}=\frac{\frac{c^2}{r}}{\sqrt{1+\left(\frac{c^2.r}{G.M}\right)^2}}\rightarrow\infty$, for curvilinear motion of a photon. This last characteristic leads to the spiral concentric path of the photon in curvilinear motion previously commented. Both field definitions meet perfectly the general equation of photon's motion for variable mass obtained in previous work [2]:

$$\frac{d^2r}{dt^2}-r.\omega^2+\frac{\mathcal{G}}{c^2}\omega^2.r^2=0 \;\Rightarrow\; \frac{d^2r}{dt^2}=0$$

In next sections we will deal with opposite cases to those treated in last two sections: we will study a photon moving away from a black hole, either in a rectilinear or curvilinear manner.

IV CAN A PHOTON ESCAPE RECTILINEAR FROM A BLACK HOLE?

Let's make the following apparently logical reasoning in order to initiate the theme: If a photon comes out from the center of a black hole (mass M is concentrated in a point) in radial motion, where Newton's definition holds, we can arrive at the conclusion that it cannot escape because its kinetic energy ($E_K=K=p.c$) would equal its potential energy ($E_P=\frac{p}{c}\frac{G.M}{r}$) at $r=r_0$ (in where the value of r_0 is $r_0=\frac{G.M}{c^2}$) and thus, it has no more energy to continue. Then, in its rectilinear trajectory it should vanish because it must stop and its speed c, changes to $c=0$, which should be impossible because constancy of the speed of light could not be violated. Let's complete this argument under the same equations: if a photon starts its radial movement from the surface of the Black Hole at a radius greater than zero but less than r_0, which means a high concentration of mass in a small volume, with enough energy, it would overcome the point of escape and black hole would emit light. Although, it is supposed that photon, coming out in a radial manner, would be condemned to stop in some point due to the permanent attraction exerted by the massive body onto the photon...

It is possible analyze this case in a more rigorous way, with the help of general equations, obtained before in [2], for applying them to a radial movement of photons against the gravitational force of attraction. We will see that for this case photon needs to start with a very high value of linear momentum in order to overcome the attraction force. Thus, we will be devoted to find out how great must be that finite and positive value of the linear momentum so that the photon becomes free from the BH, if it were possible, towards infinite with some final value finite and positive of its linear momentum. We know that according Newton's 2nd Law, and by applying the previously obtained expression of gravitational force, we find that:

$$\frac{d\mathbf{p}}{dt} = -\mathcal{F}.\mathbf{U_r} \Rightarrow \frac{d\mathbf{p}}{dt} \bullet d\mathbf{r} = d\mathbf{p} \bullet \frac{d\mathbf{r}}{dt} = -\mathcal{F}.\mathbf{U_r} \bullet d\mathbf{r} \Rightarrow dp.c = -\mathcal{F}.dr = -\frac{p}{c}.\frac{G.M}{r^2}.dr$$

$$\frac{dp}{p} = -\frac{G.M}{c^2.r^2}.dr \Rightarrow \int_{p_{BH}}^{p} \frac{dp}{p} = -\int_{r_{BH}}^{r} \frac{G.M}{c^2.r^2}.dr \quad \Rightarrow Ln\left[\frac{p}{p_{BH}}\right] = -\frac{G.M}{c^2}\left(\frac{1}{r_{BH}} - \frac{1}{r}\right) \tag{4}$$

This last relationship will allow us to obtain the equation of momentum for a photon that leaves radial and rectilinear the surface of the BH. Let $0 \le r_{BH} < r_0 = \frac{G.M}{c^2}$ be the radius of the BH. So:

$$p = p_{BH}.e^{-\left(\frac{G.M}{c^2}\right)\cdot\left(\frac{1}{r_{BH}} - \frac{1}{r}\right)} \tag{5}$$

From this last equation, we observe that if a photon takes off from the center of the BH, $r_{BH} = 0$, namely, if the BH is a mathematical point where the whole mass of the body is concentrated, the linear momentum has a null value, $p = 0$, it cannot reach any value of radius greater than zero, $r > 0$, despite of any value of p_{BH}. This means that it is not possible, for photon, to leave such theoretical mathematical point. In this particular case it cannot escape! But, if Black Hole radius is greater than zero but less than "our" version of Schwarzschild's Radius obtained in [6]: $0 < r_{BH} < r_0 = \frac{G.M}{c^2}$, momentum will have a positive and non-null value for $r = \infty$, given by:

$$p_\infty = p_{BH}.e^{-\left(\frac{G.M}{c^2}\right)\cdot\left(\frac{1}{r_{BH}}\right)} = \frac{p_{BH}}{e^{\left(\frac{G.M}{c^2.r_{BH}}\right)}} \qquad \Rightarrow \quad p_{BH} \ge p_\infty.e^{\left(\frac{G.M}{c^2.r_{BH}}\right)} \tag{6}$$

Given that $r_0 = \frac{G.M}{c^2} > r_{BH}$, then, $\frac{G.M}{c^2.r_{BH}} > 1$, and, $p_\infty = p_{BH}.e^{-\left(\frac{r_0}{r_{BH}}\right)}$, for $e = 2{,}718282$ being the Euler's Number. In this way the minimum value of p_{BH}, for $r_{BH} = r_0$, is $p_{BH} = e.p_\infty = 2{,}718282.p_\infty$. This result implies that a black hole, with a volume greater than zero, a ***real volume***, capable of emitting radial photons from its surface with a linear momentum $p_{BH} \ge p_\infty.e^{\left(\frac{r_0}{r_{BH}}\right)}$, ***can emit light!***.

We will see next, similarly to the rectilinear case, that it is also possible for a photon under some conditions escape from a black hole with a curvilinear path, parabolic or hyperbolical, and reach infinite as on previous case.

V CAN A PHOTON ESCAPE CURVILINEAR FROM A BLACK HOLE?

Let's suppose a photon leaving the surface of a REAL black hole ($r_{BH} > 0$), but in this case, describing a curvilinear movement, not circular. We will calculate how it can escape towards infinite reaching a final momentum p_∞, greater than zero but less than the momentum of takeoff p_{BH}, becoming free of the black hole, similar to the radial case. Let's take a break for remembering the classic situation of a curvilinear trajectory for the simplified case of a constant mass ($v << c$) attracted by a massive body: it requires that eccentricity e should be equal or greater than zero, $e = \frac{1}{r_0.h} - 1 > 0$. It also remembers us that the elliptical path occurs for $e < 1$, parabolic is given at $e = 1$ and hyperbolical for $e > 1$. Let us think about a similar situation for the photon, only to have a mental idea of what would happen with a photon in the next development.

The general expression of the photon's linear momentum moving curvilinear at a point located a distance r from the center of the BH, taking off of the spherical surface of the BH with radius r_{BH}, where $r > r_{BH}$, is given by [2]:

$$p = p_{BH} \cdot \left[\sqrt{\left(\frac{G.M}{c^2} . \left(\frac{1}{r_{BH}} - \frac{1}{r} \right) \right)^2 + 1} - \frac{G.M}{c^2} . \left(\frac{1}{r_{BH}} - \frac{1}{r} \right) \right] \tag{7}$$

And its value at infinite, for hyperbolic or parabolic paths, becomes:

$$p_\infty = p_{BH} \cdot \left[\sqrt{\left(\frac{G.M}{c^2} . \frac{1}{r_{BH}} \right)^2 + 1} - \frac{G.M}{c^2} . \frac{1}{r_{BH}} \right] \tag{8}$$

As it is observed $p_\infty < p_{BH}$, because the expression $\left[\sqrt{\left(\frac{G.M}{c^2} . \frac{1}{r_{BH}} \right)^2 + 1} - \frac{G.M}{c^2} . \frac{1}{r_{BH}} \right]$ is obviously less than unity. In here, we realize that in curvilinear movement of photon, the result $p_\infty < p_{BH}$ is consistent with that obtained for radial (rectilinear) in equation (6). Other aspect that can be noticed from (8) is that independently of the angle of takeoff of the surface of the BH, if this angle allows the photon escaping from BH's gravitational field, the value of linear momentum that photon can reach at infinity is the same. It depends only on the values of BH's radius, r_{BH}, and on the photon's linear momentum of takeoff of the surface of the BH, p_{BH}. Let the following simplification be recalled:

$$\frac{1}{\sqrt{\left(\frac{G.M}{c^2}.\frac{1}{r_{BH}}\right)^2+1}-\frac{G.M}{c^2}.\frac{1}{r_{BH}}}=\frac{\sqrt{\left(\frac{G.M}{c^2}.\frac{1}{r_{BH}}\right)^2+1}+\frac{G.M}{c^2}.\frac{1}{r_{BH}}}{\left(\sqrt{\left(\frac{G.M}{c^2}.\frac{1}{r_{BH}}\right)^2+1}-\frac{G.M}{c^2}.\frac{1}{r_{BH}}\right).\left(\sqrt{\left(\frac{G.M}{c^2}.\frac{1}{r_{BH}}\right)^2+1}+\frac{G.M}{c^2}.\frac{1}{r_{BH}}\right)}=$$

$$\frac{1}{\sqrt{\left(\frac{G.M}{c^2}.\frac{1}{r_{BH}}\right)^2+1}-\frac{G.M}{c^2}.\frac{1}{r_{BH}}}=\sqrt{\left(\frac{G.M}{c^2}.\frac{1}{r_{BH}}\right)^2+1}+\frac{G.M}{c^2}.\frac{1}{r_{BH}} \qquad (9)$$

We have, similar to the rectilinear case, that if a BH is capable of emitting radial photons from its surface with a linear momentum $p_{BH}=\left[\sqrt{\left(\frac{r_0}{r_{BH}}\right)^2+1}+\frac{r_0}{r_{BH}}\right].p_\infty=f(r_0,r_{BH}).p_\infty$, under certain circumstances of inclination relative to such surface, a black hole, with a volume greater than zero, a ***real volume***, can emit light! The minimum value of p_{BH}, in this case of curvilinear motion of photon for $r_{BH}=r_0$, is given by (9): $p_{BHcurv}=(2.41421356).p_\infty$. Therefore, photon to be free of a BH will be easier if it goes curvilinear than rectilinear: $p_{BHcurvil.}>p_{BHrect}=e.p_\infty=(2{,}718282).p_\infty$

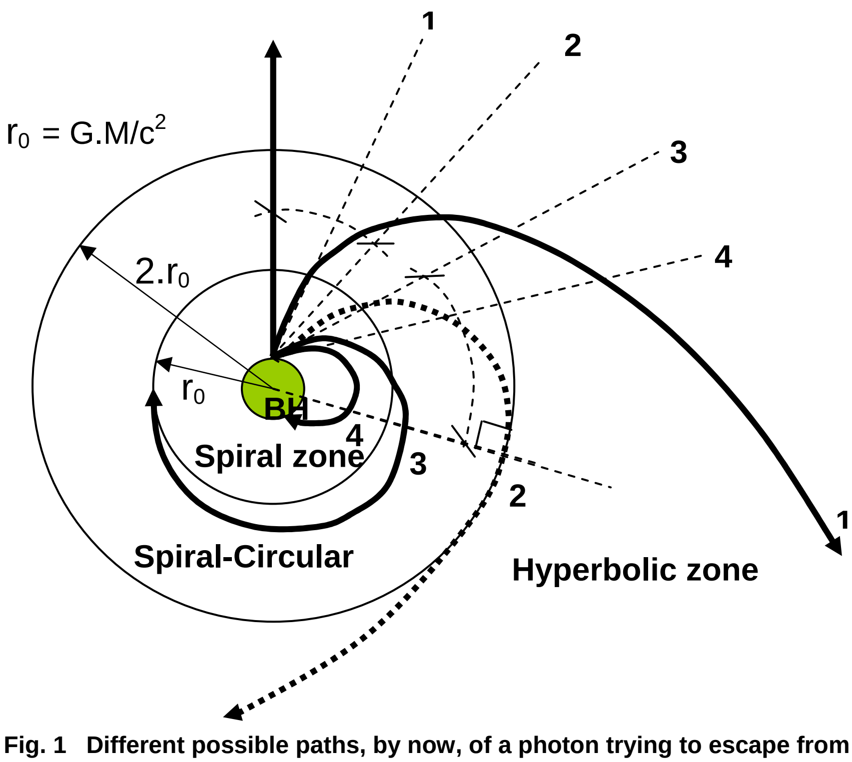

Fig. 1 Different possible paths, by now, of a photon trying to escape from a BH

From Fig. 1, it can be intuitively observed that the only possible curvilinear trajectories of a photon leaving the surface of a BH would be parabolic or hyperbolical. Other paths finish again on the BH. We can imagine that there would be two conditions in which a photon could leave such surface:

a) Its linear momentum should have a minimum value for taking off of the BH's surface. In a generic way, this value is given relative to the photon's linear momentum at infinity:

$$\frac{p_{BH}}{p_\infty} = \sqrt{\left(\frac{G.M}{c^2}.\frac{1}{r_{BH}}\right)^2 + 1} + \frac{G.M}{c^2}.\frac{1}{r_{BH}} = \sqrt{\left(\frac{r_0}{r_{BH}}\right)^2 + 1} + \frac{r_0}{r_{BH}} \qquad (10)$$

b) Additionally, photon should have some angle of inclination at its takeoff, relative to the radial line to the BH surface. Intuitively, we can establish that this angle should have a maximum value. We can imagine that while greater is the angle of takeoff of the photon, more backwards it will be will be the lift-off point. The exploration and analysis of this calculation will be done in next section.

VI MAXIMUM ANGLE RESPECT TO THE RADIAL LINE SUCH THAT A PHOTON ESCAPES FROM A BLACK HOLE

Now, let's try to find out how to calculate the maximum angle ξ, relative to the radial line at the starting point that allows a parabolic or hyperbolical trajectory of photon. One possible route involved for this maximum angle of takeoff would be when photon takes a parabolic path inside the hyperbolic zone as that indicated with a discontinuous line. In order to be familiarized with such angle formed between the radial line and the inclined line tangent to the velocity of takeoff, we are going to examine the following situation: if photon takes off perpendicular to the radial line, or what it is the same, tangential to the spherical surface of the black hole, the angle will be obviously 90 degrees, but if photon starts with some inclination, this angle will have a positive value $\xi < 90$ degrees.

Let's establish that photon leaves the BH surface of radius r_{BH}, with a minimum linear momentum p_{BH} at constant speed c, in such a way that angular momentum is preserved constant during the whole path, namely: $L = \omega_{BH}.r_{BH}^2.p_{BH} = \text{Constant}$. From the general relationship for velocities in curvilinear planar motion, the speed of photon can be divided into two components, the radial and the angular speed, such that:

$$c^2 = \left(\frac{dr}{dt}\right)^2 + (\omega.r)^2 = q^2 + (\omega.r)^2 \qquad (11)$$

Where, from Fig. 2 we can say that:

$$\xi = Arc\,tan\frac{\omega.r}{q} = Arc\,cos\,\frac{q}{c} = Arc\,sin\frac{\omega.r}{c}$$

Then we can obtain that angle ξ, at the takeoff, as the angle whose sine is given by the quotient between the product $(\omega.r)$ and the speed c:

$$\xi_{BH} = arcsin\left(\frac{\omega_{BH}.r_{BH}}{c}\right) \tag{12}$$

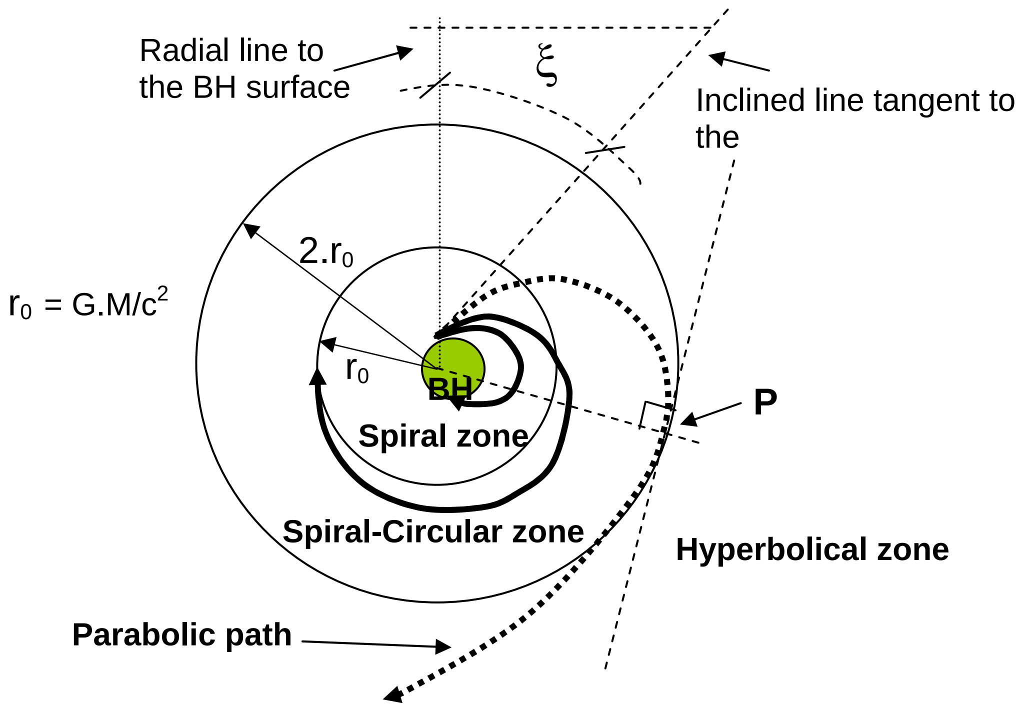

Fig. 2 Probable maximum angle ξ for a successful photon's parabolic takeoff from the BH's surface

On the other side, from Fig. 2 we can establish that at the point P where the parabolic path of photon starts, if this existed, the line tangent to the path described by the speed of photon is perpendicular to a radial line to the circle of radius $r_P = 2.r_0$, and the angle between them is 90 degrees. In this point P the radial velocity is zero and the speed of photon equals the product of angular velocity times radius. So, we can put that at this point P:

$$L = \omega_P.r_P^2.p_P = \text{Constant} \tag{13}$$

Where the following is met: $c = \omega_P.r_P$ (14)

Because radial velocity at this point P is zero, $q = 0$. Thus, applying the law of conservation of angular momentum between angular momentums at the takeoff point and at point P, the following relationship can be derived:

$$\omega_{BH}.r_{BH}^2.p_{BH} = \omega_P.r_P^2.p_P = c.r_P.p_P \quad \Rightarrow \quad \frac{\omega_{BH}.r_{BH}}{c} = \frac{r_P.p_P}{r_{BH}.p_{BH}} \tag{15}$$

In general, this relationship would be the relationship that holds for a photon taking off of the surface of the BH of radius r_{BH} with a minimum linear momentum p_{BH} and having a maximum radius r_P, where the speed of photon c is perpendicular to the radius. Obviously, if the radius of this point is less that $r_P = 2.r_0$, photon path is again striking the BH. For greater values of radius $r_P \geq 2.r_0$, namely for parabolic or hyperbolical paths, photon can escape from the gravitational zone of the BH. So, from (7), and due to $r_0 = \frac{G.M}{c^2} > r_{BH}$, and taking the frontier case $r_P = 2.r_0$, for parabolic escape we can write:

$$\frac{p_P}{p_{BH}} = \sqrt{\left(\frac{G.M}{c^2}.\left(\frac{1}{r_{BH}} - \frac{1}{r_P}\right)\right)^2 + 1} - \frac{G.M}{c^2}.\left(\frac{1}{r_{BH}} - \frac{1}{r_P}\right) = \sqrt{\left(\left(\frac{r_0}{r_{BH}} - \frac{r_0}{2.r_0}\right)\right)^2 + 1} - \left(\frac{r_0}{r_{BH}} - \frac{r_0}{2.r_0}\right)$$

$$\frac{p_P}{p_{BH}} = \sqrt{\left(\frac{r_0}{r_{BH}} - \frac{1}{2}\right)^2 + 1} - \left(\frac{r_0}{r_{BH}} - \frac{1}{2}\right) \qquad (16)$$

By introducing (15) into (14) and then into (12), we finally obtain:

$$\xi_{BH\,(max)} = arcsin\left(\frac{2.r_0}{r_{BH}}.\left[\sqrt{\left(\frac{r_0}{r_{BH}} - \frac{1}{2}\right)^2 + 1} - \left(\frac{r_0}{r_{BH}} - \frac{1}{2}\right)\right]\right) \qquad (17)$$

Given that angle is in function of these two radiuses it would be possible (?) to calculate it.

After examining obtained results of calculations based on equation (17), by giving values to $r_{BH} = k.r_0$, for the interval $0 \leq k \leq 1$, it always yields that $\frac{2.r_0}{r_{BH}}.\left[\sqrt{\left(\frac{r_0}{r_{BH}} - \frac{1}{2}\right)^2 + 1} - \left(\frac{r_0}{r_{BH}} - \frac{1}{2}\right)\right] > 1$, and because the argument of the function $arcsin$ can not be greater that unity and the relationship between depends only on radiuses, the parabolic path is not possible for a photon escaping from the BH. This leaves as unique result that the hyperbolical is the only path that photon can take for escaping from the BH, see Fig. 3.

Thus, from (15), in general the expression of the maximum angle of takeoff of the surface of the BH, relative to radial line, for an evaluation at a radius, $r_P = 2.r_0$, taking any angle at this point will be given by:

$$\xi_{BH\,(max)} = arcsin\left(\frac{\omega_p.r_p}{c}.\frac{2.r_0}{r_{BH}}.\left[\sqrt{\left(\frac{r_0}{r_{BH}} - \frac{1}{2}\right)^2 + 1} - \left(\frac{r_0}{r_{BH}} - \frac{1}{2}\right)\right]\right) \qquad (18)$$

From equation 18, it is observed that for a maximum angle of takeoff of 90 degrees ($\xi_{BH(max)} = 90°$) we can obtain the angle at $r_P = 2.r_0$ for a hyperbolical path of photon's escape. In fact, this means that:

$$90° = arcsin\left(\frac{\omega_p.r_p}{c}.\frac{2.r_0}{r_{BH}}.\left[\sqrt{\left(\frac{r_0}{r_{BH}}-\frac{1}{2}\right)^2+1}-\left(\frac{r_0}{r_{BH}}-\frac{1}{2}\right)\right]\right) \tag{19}$$

$$\Rightarrow \frac{\omega_p.r_p}{c}.\frac{2.r_0}{r_{BH}}.\left[\sqrt{\left(\frac{r_0}{r_{BH}}-\frac{1}{2}\right)^2+1}-\left(\frac{r_0}{r_{BH}}-\frac{1}{2}\right)\right]=1, \quad \Leftrightarrow \quad \frac{\omega_p.r_p}{c}=cos\varphi \tag{20}$$

Where, φ is the angle formed by tangent to the photon's path and radial line at $r_P = 2.r_0$. So,

$$\Rightarrow \frac{\omega_p.r_p}{c}=\frac{1}{\frac{2.r_0}{r_{BH}}.\left[\sqrt{\left(\frac{r_0}{r_{BH}}-\frac{1}{2}\right)^2+1}-\left(\frac{r_0}{r_{BH}}-\frac{1}{2}\right)\right]}$$

This finally gives:

$$\varphi = arccos\left(\frac{1}{2}.\left[\sqrt{\left(1-\frac{r_{BH}}{2.r_0}\right)^2+\left(\frac{r_{BH}}{r_0}\right)^2}+\left(1-\frac{r_{BH}}{2.r_0}\right)\right]\right) \tag{21}$$

The argument of (21) is always less than one for $r_{BH} = k.r_0$, in the interval $0 < k \leq 1$. This ensures that photon can leave the surface of a BH taking off even tangent to the surface of the BH. Maximum angle φ calculated for this case is 36.87 degrees, for $r_{BH} = (0{,}8)r_0$.

So, it has been demonstrated, that a photon, produced by a black hole, can leave the critical area, as in radial as in hyperbolical movement. Thus, black holes can emit light!.

It is possible, based on the previous findings, to make the following reflections or inferences about the hyperbolical trajectories of photons. Let's discuss them.

VII HOW COULD BE SEEN THE BLACK HOLES?

Photons leaving in all directions the surface of a real black hole along hyperbolical trajectories arriving at the ocular of an earth telescope will almost coincide in their orientation, giving the image of a luminous ring.

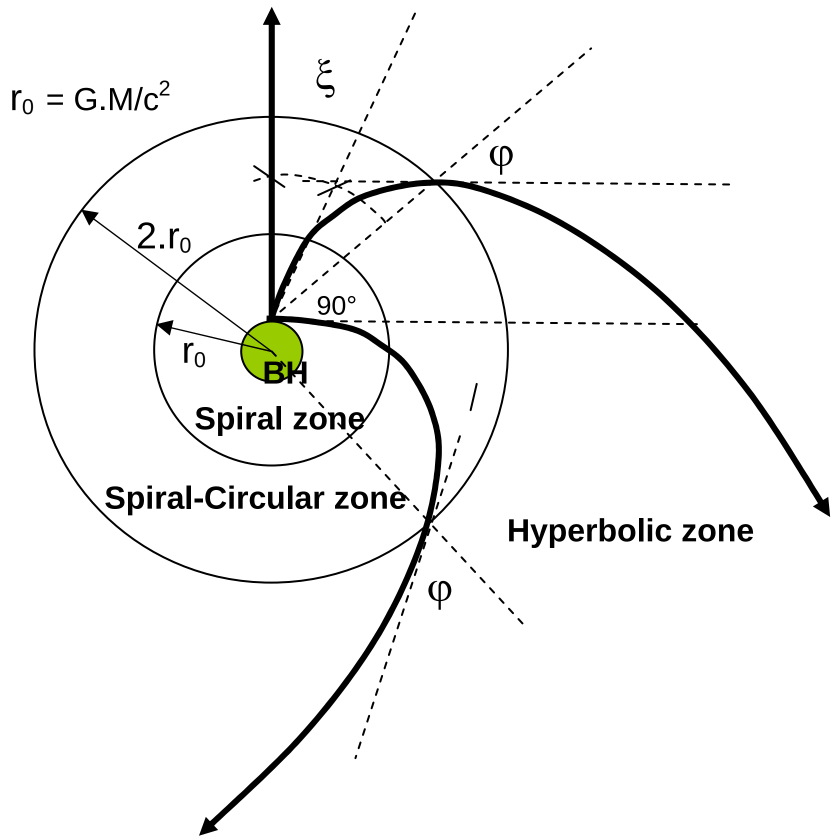

Fig. 3 Successful photon's takeoff from the BH's surface with a hyperbolical path

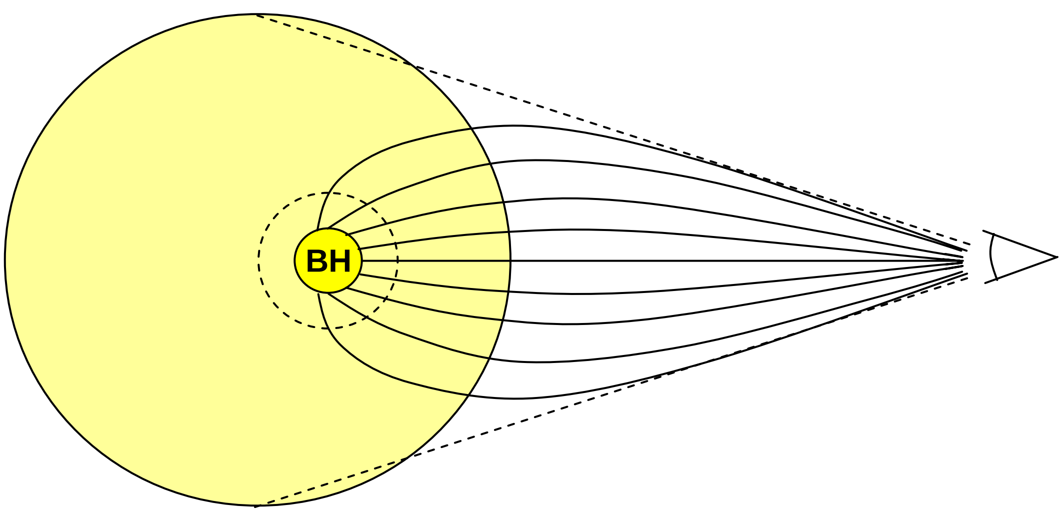

Fig. 3 Real size of the Black Hole and the magnification given to the visuals by its gravitational field

These hyperbolical photons come from all points of the surface of the black hole, included those leaving the back hemisphere of the BH. Radial photons and those almost radial, opposite to the ocular of a telescope almost coincide at a central point, with a certain density, giving the image of a luminous circular area, greater than the black hole with center at its center. The final effect of this is an image of a diffuse area gaining intensity at the center. The BH acts as an optical device, concentrating all those visual rays towards the ocular, giving a big image of the BH.

Those photons coming from the sides of the black hole, whose curvature diverges, will never reach the telescope ocular because their hyperbolical trajectories will terminate at other locations in the space.

Another effect that can be worked out as an optical effect of the BH, is the following one: Photons emitted by a star behind a black hole, attracted and passing tangentially to it, at a distance less than the critical radius r_0, will never arrive at the ocular of a earth telescope because they will be absorbed by the black hole, but if tangential photons are out of this critical orbit, they will be deflected parabolic (for deflection there is not any limitation) or hyperbolically and they could arrive at the telescope's ocular. The result is that the observer will see an image of a ring of certain luminosity depending on the star behind. These photons are almost the original ones coming from the star behind the black hole because they recover their original momentum, unlike those coming from the black hole which have photons with less momentum than they have at their takeoff points. If more stars are behind the BH more rings you will see. In any case, it will also depend upon the process that is developing in such black hole and in may be on other variables...

The following picture of the Pleiades is a good example of the previous reflections.

In general, in our opinion, some of the bodies responding to the characteristics previously discussed are real black holes ($0 < r_{BH} < r_0$) and can be easily found in the universe. Of course, if the massive body is not a black hole, it does not present rings.

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