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Vector Time and some of its Consequences in Physics

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ABSTRACT: It has been shown that time can be considered as a vector and it will be shown that its use as such introduces some variations in the shape of equations and laws in Physics. The purpose of this work is to discuss its mathematical and theoretical implications in Physics. It is worth mentioning that although this approach is relativistic it was not worked under the Einstein's General Theory of Relativity, but under the three-dimensional environment of vectorial relativity, which could be applied in all Physics.

KEYWORDS: Physics, Vectorial Relativity, Vectorial Time, Wave Equation.

I INTRODUCTION

In previous work the definition of time as vector was achieved [1] [2]. By this approach, without assumptions, we were able to derive a vectorial structure of time with **spatial components** t'_x, t'_y, t'_z, measured at O' at the origin of a moving system, in function of components t_x, t_y, t_z measured by an observer located at the origin O of an stationary system relative to the former. It is worth mentioning that Hongbao Ma in another approach briefly stated in 2004 a vectorial presentation of time depending on spatial coordinates, without referring measurements to distinct inertial observers. Namely, by a very direct procedure he obtained the components of vector time, associated to a moving point, as the relation between the components of the radio-vector of the moving point and the magnitude of its velocity, i.e., $t_x = \frac{x}{v},\ t_y = \frac{y}{v},\ t_z = \frac{z}{v}$, [3]. It is also noteworthy to mention that a rigorous presentation of time as a vector, very close to the way we have presented here can be seen in the work done by Bernard Guy [4] [5], published in 2001 and 2004, respectively.

This work is devoted to the launch of the first possible mathematical implications of vectorial time in physics. In the following section the fundamentals of time as vector are presented. In section III a definition of a new derivative-relative-to-time-components operator is presented. In Section IV Maxwell Equations are redefined using the new time operator. In section V the Wave Equation derivation by using this new time operator was achieved and in Section VI some conclusions are given.

II VECTOR TIME REVISITED

We have established in the analysis of relative motion that time is not only different for observers with distinct inertial movements, but additionally, it is forced to behave between them as a vector, whose components are scalars factors of the well-known **unit spatial vectors, i, j, k**. It is observed that this concept does not result from any assumption. Instead, the vector structure is deduced from the analysis of time's obtained expressions, for one, two, three (or more, if it were

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necessary) **spatial** dimensions. Although the case for one dimension is straightforward, it is presented as our first example (Fig.1):

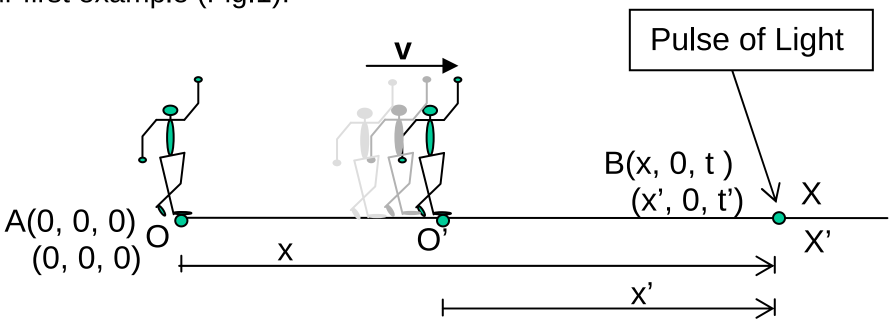

Fig. 1 One dimension

The equations that can be made and that originate the vector structure of time, from **Fig. 1**, are:

$$: x = c.t \qquad x' = k.(x - v.t) \qquad \boxed{t' = k.(t - \frac{v}{c^2}x)} \quad \Rightarrow \quad \begin{array}{l} \mathbf{t}' = k.(\mathbf{t} - \frac{v}{c^2}.\mathbf{r}) \\ \mathbf{r}' = k.(\mathbf{r} - v.\mathbf{t}) \end{array} \tag{1}$$

The two-dimensional general case, presented in **Fig. 2,** is a very clarifying example,**:**

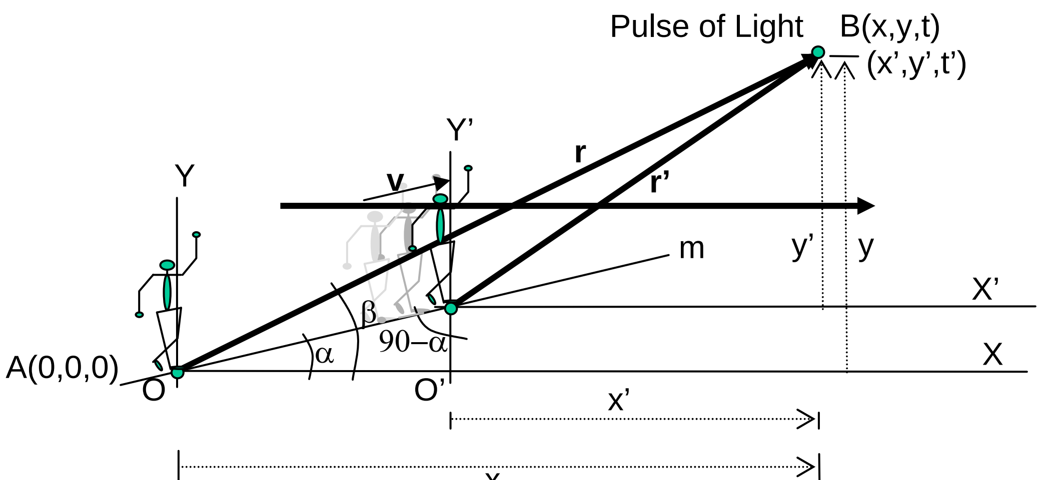

Fig. 2 Two dimensions

Referring to **Fig. 2**, when O', moving along an inclined line m, and O coincide, a light pulse is sent in any direction. By defining α, as the angle between line m, and X axis, the following equations hold:

$$\begin{array}{lcl} x^2 + y^2 = c^2.t^2 & & x' = k.(x - v.t.\cos\alpha) \\ & \text{for,} & \\ x'^2 + y'^2 = c^2.t'^2 & & y' = k.(y - v.t.\sin\alpha) \end{array} \tag{2}$$

Based on these previous relationships, by substituting, working on and grouping properly, we obtain:

$$c^2.t'^2 = x'^2 + y'^2 = k^2.[(x - v.t.\cos\alpha)^2 + (y - v.t.\sin\alpha)^2]$$
$$c^2.t'^2 = k^2.[(x^2 + y^2) + \left[v^2.(t.\cos\alpha)^2 + v^2.(t.\sin\alpha)^2\right] - 2.v.x.(t.\cos\alpha) - 2.v.y.(t.\sin\alpha)] \quad (3)$$
$$c^2.t'^2 = k^2.[c^2.(t)^2 + v^2.(t)^2 - 2.v.[x.(t.\sin\alpha) + y.(t.\sin\alpha)]]$$

Substituting: $c^2.t^2 \equiv c^2.t^2.(\sin^2\alpha + \cos^2\alpha)$, and $\text{v}^2.t^2 = \text{v}^2.\dfrac{x^2 + y^2}{c^2}$, we get:

$$c^2.t'^2 = k^2.\{[c^2.(t.\cos\alpha)^2 - 2.v.x.(t.\cos\alpha) + v^2.\frac{x^2}{c^2}] + [c^2.(t.\sin\alpha)^2 + 2.v.y.(t.\sin\alpha) + v^2.\frac{y^2}{c^2}]\}$$
$$c^2.t'^2 = k^2.[(c.t.\cos\alpha - \frac{v}{c}.x)^2 + (c.t.\sin\alpha - \frac{v}{c}.y)^2] = c^2.k^2.[(t.\cos\alpha - \frac{v}{c^2}.x)^2 + (t.\sin\alpha - \frac{v}{c^2}.y)^2] \quad (4)$$

From the last relationship, the following expression for time is obtained:

$$t'^2 = k^2.[(t.\cos\alpha - \frac{v}{c^2}.x)^2 + (t.\sin\alpha - \frac{v}{c^2}.y)^2] \quad (6)$$

By observing carefully the right hand side of the previous expression, it reminds us the module of a vector. Thus, as it is suggested, the previous modular expression can be re-organized into its corresponding two-dimensional vectorial structure, in the following way:

$$\mathbf{t'} = k.\left[\left(t.\cos\alpha - \frac{v}{c^2}.x.\right)\mathbf{i} + \left(t.sin\alpha - \frac{v}{c^2}.y\right).\mathbf{j}\right] = k.\left[t.\cos\alpha.\mathbf{i} - \frac{v}{c^2}.x.\mathbf{i} + t.sin\alpha.\mathbf{j} - \frac{v}{c^2}.y.\mathbf{j}\right]$$
$$\mathbf{t'} = k.\left[(t.\cos\alpha.\mathbf{i} + t.sin\alpha.\mathbf{j}) - \frac{v}{c^2}.(x.\mathbf{i} + y.\mathbf{j})\right] = k.\left[(t_x.\mathbf{i} + t_y.\mathbf{j}) - \frac{v}{c^2}.(x.\mathbf{i} + y.\mathbf{j})\right] \quad (7)$$

Thus, by defining: $\begin{Bmatrix} t_x = t.\cos\alpha \\ t_y = t.\sin\alpha \\ \mathbf{t} = t_x\mathbf{i} + t_y\mathbf{j} \end{Bmatrix}$ and $\begin{Bmatrix} t'_x = k.\left(t_x - \dfrac{v}{c^2}.x\right) \\ t'_y = k.\left(t_y - \dfrac{v}{c^2}.y\right) \end{Bmatrix} \Rightarrow \begin{Bmatrix} \mathbf{t'} = k.\left(\mathbf{t} - \dfrac{v}{c^2}.\mathbf{r}\right) \\ \mathbf{r'} = k.(\mathbf{r} - v.\mathbf{t}) \end{Bmatrix}$ (8)

It can be realized that this vector structure of time can be easily obtained for any number of dimensions by repeating this same procedure. For instance:

In the three-dimensional case, see **Fig. 3**, the following relationships hold:

$$\begin{matrix} x^2 + y^2 + z^2 = c^2.t^2 & x' = k.(x - v.t.\cos\alpha.\cos\beta) & t_x = t.\cos\alpha.\cos\beta \\ x'^2 + y'^2 + z'^2 = c^2.t'^2 & y' = k.(y - v.t.\sin\alpha.\cos\beta) & t_y = t.\sin\alpha.\cos\beta \\ & z' = k.(z - v.t.\sin\beta) & t_z = t\sin\beta \end{matrix} \quad (9)$$

Following a similar procedure to that previously used is obtained again the familiar vector structure expression of time for three (or for any number of) dimensions:

$$\boxed{t'^2 = k^2.[(t_x - \frac{v}{c^2}.x)^2 + (t_y - \frac{v}{c^2}.y)^2 + (t_z - \frac{v}{c^2}.z)^2]} \quad \Rightarrow \quad \begin{aligned} \mathbf{t}' &= k.(\mathbf{t} - \frac{v}{c^2}.\mathbf{r}) \\ \mathbf{r}' &= k.(\mathbf{r} - v.\mathbf{t}) \end{aligned} \qquad (10)$$

All these results lead consistently to consider the behavior of time as a vector when it is referred to observers located in systems with different inertial movements.

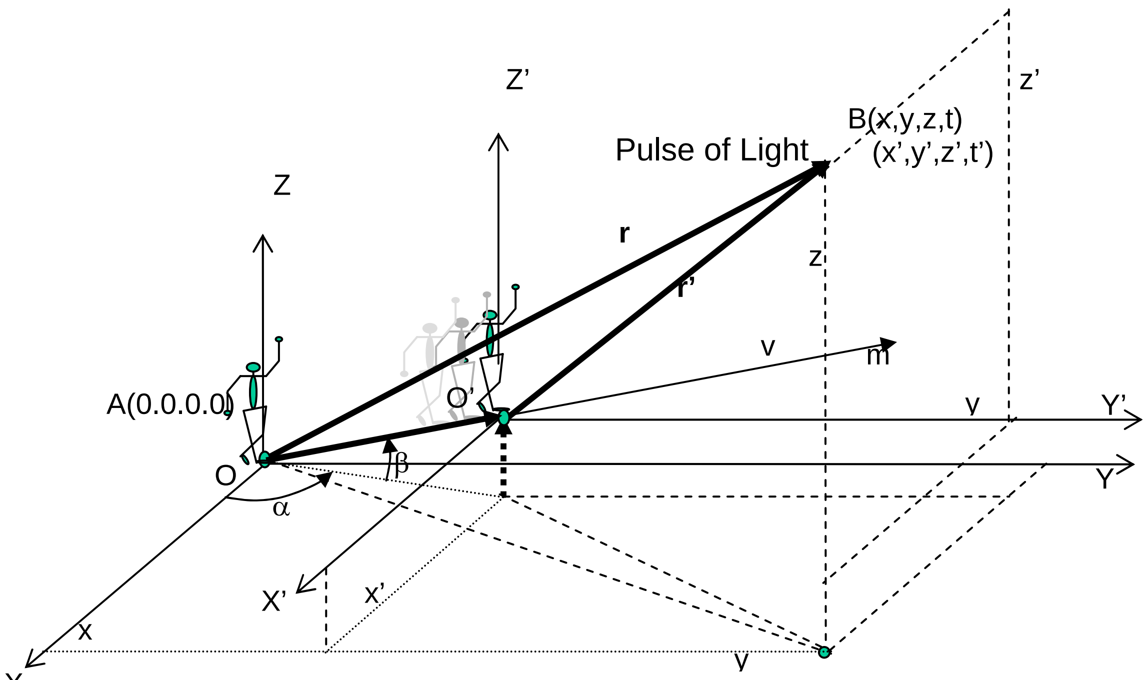

Fig. 3 **Three dimensions**

Obviously, the invariance of the space-time interval of the Special Theory of Relativity is preserved for **any number of dimensions,** i.e.: For $k = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$ and substituting general coordinate components in the expression of the space-time interval, $c^2.t'^2 - r'^2$:

$$c^2.t'^2 - r'^2 = \sum_{j=1}^{N} c^2.k^2.\left(t_j - \frac{v}{c^2}.x_j\right)^2 - k^2.\sum_{j=1}^{N}\left(x_j - v.t_j\right)^2 = k^2.\sum_{j=1}^{N}\left[\left(c.t_j - \frac{v}{c}.x_j\right)^2 - \left(x_j - v.t_j\right)^2\right] =$$

$$= k^2.\sum_{j=1}^{N}\left[\left(c^2.t_j^2 - 2.v.t_j.x_j + \frac{v^2}{c^2}.x_j^2\right) - \left(x_j^2 - 2.v.t_j.x_j + v^2.t_j^2\right)\right] = k^2.\sum_{j=1}^{N}\left[c^2.t_j^2 + \frac{v^2}{c^2}.x_j^2 - x_j^2 - v^2.t_j^2\right]$$

$$= k^2.\sum_{j=1}^{N}\left[c^2.t_j^2 + \frac{v^2}{c^2}.x_j^2 - x_j^2 - v^2.t_j^2\right] = k^2.\sum_{j=1}^{N}\left[c^2.t_j^2 - v^2.t_j^2 + \frac{v^2}{c^2}.x_j^2 - x_j^2\right] =$$
$$= k^2.\left(1 - \frac{v^2}{c^2}\right).\sum_{j=1}^{N}\left(c^2.t_j^2 - x_j^2\right) = \sum_{j=1}^{N}\left(c^2.t_j^2 - x_j^2\right) = \sum_{j=1}^{N}\left(c^2.t_j^2\right) - \sum_{j=1}^{N}\left(x_j^2\right) \Rightarrow \boxed{c^2.t'^2 - r'^2 = c^2.t^2 - r^2} \qquad (11)$$

So, the vector character of time is not the result of any hypothesis; it comes directly from observing vector properties clearly present inside transformations relating measurements of both inertial observers. It can also be observed, from an epistemological point of view, that time as vector forms its direction by taking it from the vector velocity v of the moving system O', leaving such parameter with a scalar character and functioning as a scaling factor. This can be understood due to both observers are on the same inclined line, which will imply the scalar character of v. Another epistemological characteristic of vector time is <u>its dependence on coordinates</u> x, y and z, which means that **it is not an independent vector**, a characteristic that appears as remarkable because differs to that of four dimensions (time as a fourth independent dimension) introduced by Einstein. This also means that we are still working in three spatial dimensions in this study, and that magnitudes can continue being defined as in classical physics, but with a modern and relativistic view. From observing these **results** we can develop the following definition of time: Time is forced to behave as a vector with spatial components in each coordinate, when it appears inside the VLT, but it can appear behaving as a scalar when it is not an element of a transformation such as VLT in the way we always have known it: as a sequential meter of events. But moreover, time can also be considered as a vector in the natural way it was referred to previously in [4]. In accordance with this idea and it perfectly applies to our work, Hongbao Ma says: "*this three dimensional time concept is obtained from the mathematical conception rather than the ontological existence. Mathematical results are at the epistemological level*" [4].

III PARTIAL DERIVATIVE RELATIVE TO TIME-VECTOR

Let's try to find out how we can do that. For example by remember the definition of the operator ∇:

$$\nabla \equiv \left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}\right) \qquad (12)$$

Its relation with the radio-vector of position $\mathbf{r}$ of a generic moving point in space can be put as::

$$\mathbf{r} = x.\mathbf{i} + y.\mathbf{j} + z.\mathbf{k} \quad \Rightarrow \quad r = \sqrt{x^2 + y^2 + z^2} \quad \Rightarrow \quad \frac{\partial r}{\partial x} = \frac{x}{r};\ \frac{\partial r}{\partial y} = \frac{y}{r};\ \frac{\partial r}{\partial z} = \frac{z}{r}; \qquad (13)$$

Developing partial derivatives of components x, y, z, depending only on r and t, we have:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial}{\partial t}\frac{\partial t}{\partial x};\ \frac{\partial}{\partial y} = \frac{\partial}{\partial r}\frac{\partial r}{\partial y} + \frac{\partial}{\partial t}\frac{\partial t}{\partial y};\ \frac{\partial}{\partial z} = \frac{\partial}{\partial r}\frac{\partial r}{\partial z} + \frac{\partial}{\partial t}\frac{\partial t}{\partial z}; \quad \text{For ,} \ \frac{\partial t}{\partial x} = \frac{\partial t}{\partial y} = \frac{\partial t}{\partial z} = 0 \qquad (14)$$

So, the operator $\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$ can be expressed in function of the radio-vector as:

$$\nabla = \frac{\partial}{\partial r}\frac{\partial r}{\partial x}\mathbf{i} + \frac{\partial}{\partial r}\frac{\partial r}{\partial y}\mathbf{j} + \frac{\partial}{\partial r}\frac{\partial r}{\partial z}\mathbf{k} = \frac{\partial}{\partial r}\frac{x}{r}\mathbf{i} + \frac{\partial}{\partial r}\frac{y}{r}\mathbf{j} + \frac{\partial}{\partial r}\frac{z}{r}\mathbf{k} = \frac{\partial}{\partial r}\frac{\mathbf{r}}{r} \quad \Rightarrow \quad \nabla = \frac{\partial}{\partial r}\frac{\mathbf{r}}{r} \qquad (15)$$

And also with the squared spatial operator, the Laplacian:

$$\nabla \bullet \nabla = \left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}\right) \bullet \left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}\right) = \nabla^2 = \frac{\partial^2}{\partial r^2}\left(\frac{\mathbf{r}}{r} \bullet \frac{\mathbf{r}}{r}\right)$$
$$\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) = \frac{\partial^2}{\partial r^2} \qquad (16)$$

By repeating the procedure to obtain the relationship between the **space** operator ∇ and the radio-vector $\mathbf{r}$, but referred to the **time** operator $\overline{\nabla}$, and the vector time $\mathbf{t}$, we obtain:

$$\overline{\nabla} \equiv \left(\frac{\partial}{\partial t_x}\mathbf{i} + \frac{\partial}{\partial t_y}\mathbf{j} + \frac{\partial}{\partial t_z}\mathbf{k}\right) \qquad (17)$$

$$\mathbf{t} = t_x.\mathbf{i} + t_y.\mathbf{j} + t_z.\mathbf{k} \quad \Rightarrow \quad t = \sqrt{t_x^2 + t_y^2 + t_z^2} \quad \Rightarrow \quad \frac{\partial t}{\partial t_x} = \frac{t_x}{t};\ \frac{\partial t}{\partial t_y} = \frac{t_y}{t};\ \frac{\partial t}{\partial t_z} = \frac{t_z}{t}; \qquad (18)$$

Thus, each one of the components of the operator $\overline{\nabla}$ can be represented as depending on both vectors. Let's arrive at such equations depending only on r and t. For this, we will develop the expressions depending on the components t_x, t_y, t_z:

$$\frac{\partial}{\partial t_x} = \frac{\partial}{\partial r}\frac{\partial r}{\partial t_x} + \frac{\partial}{\partial t}\frac{\partial t}{\partial t_x};\ \frac{\partial}{\partial t_y} = \frac{\partial}{\partial r}\frac{\partial r}{\partial t_y} + \frac{\partial}{\partial t}\frac{\partial t}{\partial t_y};\ \frac{\partial}{\partial t_z} = \frac{\partial}{\partial r}\frac{\partial r}{\partial t_z} + \frac{\partial}{\partial t}\frac{\partial t}{\partial t_z};\ \text{Where,}\ \frac{\partial r}{\partial t_x} = \frac{\partial r}{\partial t_y} = \frac{\partial r}{\partial t_z} = 0$$

So, the vectorial-time operator $\overline{\nabla} = \frac{\partial}{\partial t_x}\mathbf{i} + \frac{\partial}{\partial t_y}\mathbf{j} + \frac{\partial}{\partial t_z}\mathbf{k}$ can be expressed in function of the time-vector as:

$$\overline{\nabla} = \frac{\partial}{\partial t}\frac{\partial t}{\partial t_x}\mathbf{i} + \frac{\partial}{\partial t}\frac{\partial t}{\partial t_y}\mathbf{j} + \frac{\partial}{\partial t}\frac{\partial t}{\partial t_z}\mathbf{k} = \frac{\partial}{\partial t}\frac{t_x}{t}\mathbf{i} + \frac{\partial}{\partial t}\frac{t_y}{t}\mathbf{j} + \frac{\partial}{\partial t}\frac{t_z}{t}\mathbf{k} = \frac{\partial}{\partial t}\frac{\mathbf{t}}{t} \quad \Rightarrow \quad \overline{\nabla} = \frac{\partial}{\partial t}\frac{\mathbf{t}}{t} \qquad (19)$$

$$\left|\overline{\nabla}\right| = \sqrt{\frac{\partial}{\partial t}\frac{\mathbf{t}}{t} \bullet \frac{\partial}{\partial t}\frac{\mathbf{t}}{t}} = \frac{\partial}{\partial t}; \qquad \text{Also,}\ \left|\overline{\nabla}\right| = \overline{\nabla} \bullet \frac{\mathbf{t}}{t} = \frac{\partial}{\partial t}\frac{\mathbf{t}}{t} \bullet \frac{\mathbf{t}}{t} = \frac{\partial}{\partial t} \quad \Rightarrow \quad \left|\overline{\nabla}\right| = \frac{\partial}{\partial t} \qquad (20)$$

And also with the squared vectorial-time operator, which we will call it hereon, *the T-Laplacian*:

$$\overline{\nabla}\bullet\overline{\nabla}=\left(\frac{\partial}{\partial t_x}\mathbf{i}+\frac{\partial}{\partial t_y}\mathbf{j}+\frac{\partial}{\partial t_z}\mathbf{k}\right)\bullet\left(\frac{\partial}{\partial t_x}\mathbf{i}+\frac{\partial}{\partial t_y}\mathbf{j}+\frac{\partial}{\partial t_z}\mathbf{k}\right)=\overline{\nabla}^2=\frac{\partial^2}{\partial t^2}\left(\frac{\mathbf{t}}{t}\bullet\frac{\mathbf{t}}{t}\right)$$

$$\overline{\nabla}^2=\left(\frac{\partial^2}{\partial {t_x}^2}+\frac{\partial^2}{\partial {t_y}^2}+\frac{\partial^2}{\partial {t_z}^2}\right)=\frac{\partial^2}{\partial t^2}\quad\Leftrightarrow\quad\overline{\nabla}^2=\left|\overline{\nabla}\right|^2 \qquad (21)$$

IV CONSEQUENCES ON MAXWELL EQUATIONS

Concept	***Differential Form***	***Integral Form***	
Gauss Law for electric charges.	$\nabla\bullet\mathbf{D}=\rho$	$\oint_S\mathbf{D}\bullet d\mathbf{A}=\int_V\rho.dV$	(22)
($\nabla\bullet$): Spatial Divergence Operator; ($\mathbf{D}$): Electric Displacement; (ρ): Electric Charge Density			
Gauss Law for magnetism.	$\nabla\bullet\mathbf{B}=0$	$\oint_S\mathbf{B}\bullet d\mathbf{A}=0$	(23)
($\mathbf{B}$): Magnetic Flux Density			
Faraday's Law (Classic).	$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial \text{t}}$	$\oint_C\mathbf{E}\bullet d\mathbf{l}=-\int_S\frac{\partial\mathbf{B}}{\partial t}\bullet d\mathbf{A}$	(24)
($\nabla\times$): Spatial Curl Operator; ($\mathbf{E}$): Electric Field			
Faraday's Law (Vectorial Relativity).	$\nabla\times\mathbf{E}=-\left|\overline{\nabla}\right|\mathbf{B}$	$\oint_C\mathbf{E}\bullet d\mathbf{l}=-\int_S\left|\overline{\nabla}\right|\mathbf{B}\bullet d\mathbf{A}$	(25)
($\left|\overline{\nabla}\right|=\frac{\partial}{\partial t}$): Time Scalar Derivative Operator;		$\mathbf{B}=\mu_0.\mathbf{H}$ $\quad\mathbf{D}=\varepsilon_0.\mathbf{E}$	
Ampere's Law (Classic).	$\nabla\times\mathbf{H}=\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}$	$\oint_C\mathbf{H}\bullet d\mathbf{l}=\int_S\mathbf{J}\bullet d\mathbf{A}+\int_S\frac{\partial\mathbf{D}}{\partial t}\bullet d\mathbf{A}$	(26)
($\mathbf{H}$): Magnetic Field; ($\mathbf{J}$): Current Density			
Ampere's Law (Vect. Rel.).	$\nabla\times\mathbf{H}=\mathbf{J}+\left|\overline{\nabla}\right|\mathbf{D}$	$\oint_C\mathbf{H}\bullet d\mathbf{l}=\int_S\mathbf{J}\bullet d\mathbf{A}+\int_S\left|\overline{\nabla}\right|\mathbf{D}\bullet d\mathbf{A}$	(27)

V APPLICATIONS. WAVE EQUATION.

As we know, the Equation of an Electromagnetic Wave that displaces in free space can be directly deduced from Maxwell's Equations. For example: By taking the curl of Faraday's Law:

$$\nabla\times(\nabla\times\mathbf{E})=-\nabla\times\frac{\partial\mathbf{B}}{\partial \text{t}}=-\frac{\partial(\nabla\times\mathbf{B})}{\partial \text{t}}=-\mu_0.\frac{\partial(\nabla\times\mathbf{H})}{\partial \text{t}} \qquad (28)$$

And substituting Ampere's Law for a free space in where $\mathbf{J}=0$ and $\nabla\bullet\mathbf{D}=\nabla\bullet\varepsilon_0.\mathbf{E}=0$:

$$\nabla\times(\nabla\times\mathbf{E})=-\mu_0.\frac{\partial(\nabla\times\mathbf{H})}{\partial\mathrm{t}}=-\mu_0.\frac{\partial\left(\frac{\partial\mathbf{D}}{\partial t}\right)}{\partial\mathrm{t}}=-\mu_0.\frac{\partial\left(\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}\right)}{\partial\mathrm{t}}=-\mu_0.\varepsilon_0.\frac{\partial^2\mathbf{E}}{\partial t^2}=-c^2.\frac{\partial^2\mathbf{E}}{\partial t^2}$$

Using the vectorial identity: $\mathbf{A}\times(\mathbf{B}\times\mathbf{C})=\mathbf{B}(\mathbf{A}\bullet\mathbf{C})-\mathbf{C}(\mathbf{A}\bullet\mathbf{B})$:

$$\nabla\times(\nabla\times\mathbf{E})=\nabla(\nabla\bullet\mathbf{E})-\mathbf{E}(\nabla\bullet\nabla)=-c^2.\frac{\partial^2\mathbf{E}}{\partial t^2}\quad\Rightarrow\quad-\mathbf{E}(\nabla\bullet\nabla)=-\nabla^2\mathbf{E}=-c^2.\frac{\partial^2\mathbf{E}}{\partial t^2}$$

$$\boxed{\nabla^2\mathbf{E}-c^2.\frac{\partial^2\mathbf{E}}{\partial t^2}=0} \tag{29}$$

A first way of demonstrating the validity of the Vectorial Relativity approach in the derivation of the wave equation is by substituting the spatial Laplacian and the T-Laplacian by their equivalents, $\nabla^2=\frac{\partial^2}{\partial r^2}$ and $\overline{\nabla}^2=\frac{\partial^2}{\partial t^2}$, respectively, previously obtained. Namely, the classical wave equation $\nabla^2\mathbf{E}-c^2.\frac{\partial^2\mathbf{E}}{\partial t^2}=0$, can be directly corresponded to the new version in Vectorial Relativity:

$$\boxed{\nabla^2\mathbf{E}-c^2.\overline{\nabla}^2\mathbf{E}=0}. \tag{30}$$

But may be it is better to obtain it in the same vectorial way as we did before in order to observe the vectorial properties of time as vector and the limitations of its use (as for example, it seems that the Time Divergence Operator . By taking the curl of Faraday's Law:

$$\nabla\times(\nabla\times\mathbf{E})=-\nabla\times\left|\overline{\nabla}\right|\mathbf{B}=-\left|\overline{\nabla}\right|(\nabla\times\mu_0.\mathbf{H})=-\mu_0.\left|\overline{\nabla}\right|\left(\mathbf{J}+\left|\overline{\nabla}\right|\mathbf{D}\right) \tag{31}$$

Substituting in Ampere's Law free space conditions in where $\mathbf{J}=0$ and $\nabla\bullet\mathbf{E}=0$. And using the vectorial identity , $\nabla\times(\nabla\times\mathbf{E})=\nabla(\nabla\bullet\mathbf{E})-\mathbf{E}(\nabla\bullet\nabla)$, we finally have:

$$\nabla(\nabla\bullet\mathbf{E})-\mathbf{E}(\nabla\bullet\nabla)=-\mu_0\left|\overline{\nabla}\right|^2\mathbf{D}=-\mu_0.\varepsilon_0\overline{\nabla}^2\mathbf{E}$$

$$-\nabla^2\mathbf{E}=-\mu_0.\varepsilon_0.\overline{\nabla}^2\mathbf{E}\qquad\Rightarrow\qquad\nabla^2\mathbf{E}-c^2.\overline{\nabla}^2\mathbf{E}=0 \tag{32}$$

This implies that the wave equation can be written like each of the following valid expressions:

$$\frac{\partial^2\mathbf{E}}{\partial x^2}+\frac{\partial^2\mathbf{E}}{\partial y^2}+\frac{\partial^2\mathbf{E}}{\partial z^2}-c^2.\left(\frac{\partial^2\mathbf{E}}{\partial t_x{}^2}+\frac{\partial^2\mathbf{E}}{\partial t_y{}^2}+\frac{\partial^2\mathbf{E}}{\partial t_z{}^2}\right)=0 \tag{33}$$

$$\nabla^2\mathbf{E}-c^2.\overline{\nabla}^2\mathbf{E}=0 \tag{34}$$

$$\frac{\partial^2\mathbf{E}}{\partial r^2} - c^2.\frac{\partial^2\mathbf{E}}{\partial t^2} = 0 \qquad (35)$$

Or its cross-combinations:

$$\frac{\partial^2\mathbf{E}}{\partial x^2} + \frac{\partial^2\mathbf{E}}{\partial y^2} + \frac{\partial^2\mathbf{E}}{\partial z^2} - c^2.\overline{\nabla}^2\mathbf{E} = 0 \qquad (36)$$

$$\frac{\partial^2\mathbf{E}}{\partial x^2} + \frac{\partial^2\mathbf{E}}{\partial y^2} + \frac{\partial^2\mathbf{E}}{\partial z^2} - c^2.\frac{\partial^2\mathbf{E}}{\partial t^2} = 0 \qquad (37)$$

$$\nabla^2\mathbf{E} - c^2.\left(\frac{\partial^2\mathbf{E}}{\partial t_x{}^2} + \frac{\partial^2\mathbf{E}}{\partial t_y{}^2} + \frac{\partial^2\mathbf{E}}{\partial t_z{}^2}\right) = 0 \qquad (38)$$

$$\nabla^2\mathbf{E} - c^2.\frac{\partial^2\mathbf{E}}{\partial t^2} = 0 \qquad (39)$$

$$\frac{\partial^2\mathbf{E}}{\partial r^2} - c^2.\left(\frac{\partial^2\mathbf{E}}{\partial t_x{}^2} + \frac{\partial^2\mathbf{E}}{\partial t_y{}^2} + \frac{\partial^2\mathbf{E}}{\partial t_z{}^2}\right) = 0 \qquad (41)$$

$$\frac{\partial^2\mathbf{E}}{\partial r^2} - c^2.\overline{\nabla}^2\mathbf{E} = 0 \qquad (42)$$

VI CONCLUSIONS

From a mathematical point of view we consider that vector time introduced through partial derivatives in Maxwell's equations, has not produced severe consequences for physics since we could have hoped, until now. Nevertheless, it is left for further research the definition and meaning of the time divergence operator of some vector field $\mathbf{Q}$, ($\overline{\nabla} \bullet \mathbf{Q}$), and its Time Curl Operator, ($\overline{\nabla} \times \mathbf{Q}$), if these ones existed, and their applications in Physics, if it were possible.

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