JVR

Journal of Vectorial Relativity
JVR **2** (2007) **1** 48-52

Letter

Schwarzschild Radius in Vectorial Relativity

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ABSTRACT: In previous work it was demonstrated that the correct equation that governs gravitational effect of a massive body, considered it with a fixed and constant mass, on a photon moving at velocity c and momentum p, which has a variable mass in its curved path given by $m=\frac{p}{c}$, becomes: $\frac{d^2r}{dt^2}-r.\omega^2+\frac{\mathrm{G}}{c^2}\omega^2.r^2=0$, where the value of the gravitational field G exerted by the massive body on the variable mass of photon was found to be: $\mathrm{G}=\frac{\frac{2.G.M}{r^2}}{\frac{p}{p_0}+\frac{p_0}{p}}$, denoting by p_0 the constant value of the linear momentum for a photon attracted by the massive body, at its nearest point (r_0) and p the generic value of the linear momentum of the photon at any other point. One of the reasons of the success of Einstein's General Theory of Relativity (GTR) is that it allows calculating planet's precession (rotation of the elliptical path axis with time, i.e.: Mercury Precession). This fact, that its occurrence has been experimentally observed, is not accounted by classic Kepler's Laws because it is applicable only to constant masses. In this work, as a direct result of only considering mass as a variable inside equations and applying known and accepted physical laws, it is shown that precession appears as one of the natural outcomes, preserving speed of light constant. Conversely, one of the criticisms to GTR is that Einstein's Field Equations solutions depend on considered curvature of the space-time (Schwarzschild, Reissner-Nordström, Kerr, Friedman-Lemaitre, etc.), and that attracted mass is considered as constant (!), and what is worst, in some of these solutions speed of a photon going radial towards a massive body can be distinct of that of light.

KEYWORDS: Universal Gravitation, Kepler Laws, Schwazschild Radius, Vectorial Relativity, STR, GTR.

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I. INTRODUCTION

This work is a continuation of previous one on Energy [1] and Gravitation [2], also reviewed in this issue [3]. This work is devoted to illustrate inconsistencies found in some solutions of the Einstein Field Equations (EFE). At the end of this work are presented some conclusions.

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II. SCHWARZSCHILD RADIUS.

In this section we are going to dedicate these first paragraphs to briefly discuss Schwarzschild solution to the Einstein's Field Equations (EFE) and some of its results or predictions, because although each one of them have been confirmed at a "0.xx" percent of accuracy by the modern tests of GTR, some of their results are not so convincing as to allow accepting them without express our disturbance.

In fact, in the deduction of the Schwarzschild metric you don't observe the effect of a moving mass in the sense of its variation relative to its velocity and how it modifies the equation of motion. Or, where is, inside the Schwarzschild development, that major outcome of the Special Theory of Relativity as it was the variation of mass with its speed v and that of the speed of light c?. Or, usually one finds phrases starting this treatment with "We shall not give the reasons why Schwarzschild arrived at the form ….".

Under such environment the Schwarzschild metric that defines the space-time outside a spherical massive body of mass M , meeting EFE conditions, is given by:

$$ds^2 = -c^2\left(1-\frac{r_S}{r}\right)dt^2 + \left(1-\frac{r_S}{r}\right)^{-1}dr^2 + r^2.d\theta^2 + r^2.\sin^2\theta.d\phi^2 \quad \text{for} \quad \boxed{r_S = \frac{2.G.M}{c^2}} \tag{1}$$

By considering null geodesics, radial light photons where $d\theta = d\phi = 0$, Schwarzschild equation reduces to:

$$0 = -c^2\left(1-\frac{r_S}{r}\right)dt^2 + \left(1-\frac{r_S}{r}\right)^{-1}dr^2 \;\Rightarrow\; c^2dt^2 = \frac{dr^2}{\left(1-\frac{r_S}{r}\right)^2} \;\Rightarrow\; \frac{dr}{dt} = \pm c.\left(1-\frac{r_S}{r}\right) \tag{2}$$

This is the expression of photon's radial velocity. What does it mean?. We can grasp that when photon approaches to the massive body, $r \to r_S$, its radial velocity decreases, and if that whole mass of massive body is concentrated in a very little sphere with a radius less than r_S , then, when radius continues diminishing until it reaches $r = r_S$, photon stops (?). So, the speed of photon is not constant, is it?. On the contrary, if photon goes in the opposite sense, r increases until it goes infinite and radial velocity finally gets its "constant" value c . I don't agree this kind of results because, in my opinion, it contradicts the very basis and fundamentals of Relativity, for example, the postulate of the constancy of light speed. Really, I am convinced of its correctness, because experimental measurements done along all twentieth and twenty first centuries have confirmed that such postulate is correct. It is important to say that the previous referred contradiction could be predictable, due to the existence in the Schwarzschild metric of a factor different from unity, as it was previously shown, multiplying the term $c^2.dt^2$.

On the other hand, in previous issue of JVR [1] and in the current one [2], it was demonstrated from the very beginnings that analysis of motion of a particle gravitationally attracted by a massive body leads to simple equations, either for the case of constant masses or for those of variable masses. For instance, for the special case of constant radius (circular motion) focused from either our relativistic point of view (Vectorial Relativity) that considers a moving variable mass attracted by another considered fixed, or by applying Newton's Universal Gravitation law, applicable only for constant masses, both views holds. So, for constant radius, and accordingly, speed remains constant, same result readily arises:

A) Under our point of view, see reference [2] equation (23), but substituting the constant values of radius and speed of photon for this case, the result, in fact is:

$$\frac{d^2r_0}{dt^2} - r_0.\omega_0^2 + \frac{G.M}{c^2}\omega_0^2 = 0 \quad \Rightarrow \quad \textbf{for} \quad \frac{d^2r_0}{dt^2} = 0 \;\Rightarrow\; r_0.\omega_0^2 = \frac{G.M}{c^2}.\omega_0^2 \Rightarrow \quad r_0 = \frac{G.M}{c^2} \qquad (3)$$

B) With Newtonian equations, remember the two scalars equations coming from the Vectorial equation (1) in referred reference [2]:

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = -\frac{G.M}{r^2} \quad \Rightarrow \quad \frac{d^2r}{dt^2} - r.\omega^2 = -\frac{G.M}{r^2}$$

Substituting, it readily follows as expected, the same result:

$$\frac{d^2r_0}{dt^2} - r_0.\omega_0^2 = -\frac{G.M}{r_0^2} \quad \Rightarrow \quad \textbf{for} \quad \frac{d^2r_0}{dt^2} = 0 \;\; \textbf{and} \;\; r_0.\omega_0 = c \;\Rightarrow\; r_0^2\omega_0^2 = \frac{G.M}{r_0} \quad \Rightarrow \quad r_0 = \frac{G.M}{c^2} \qquad (4)$$

In our opinion this is a logical and consistent result for this special case, because, under the same conditions of constant velocity or constant mass, our approach reduces to Newtonian prediction which is our best reference, because photon of light has a mass given by $m = \frac{p}{c} = \frac{E}{c^2} = \frac{h.\gamma}{c^2}$, in this case constant, and as any mass it has a similar expression to that given by Newton approach. On the other hand, under same conditions, in spite of previous reasoning Schwarzschild's result does not reduce to Newtonian value.

Let's take another case of "success" of Schwarzschild metric: bending of light. This is the deviation suffered by a photon in its free and rectilinear trajectory after it falls in the field of attraction of a massive body. The resulting formula of the angle deviation of photon, given by Schwarzschild analysis governing bending of light is:

$$\Delta\varphi = \frac{4.G.M}{r_S.c^2} \qquad (5)$$

This is two times the value given by Newtonian gravity. However, although Newtonian equations predicts a bending too, half the previous Schwarzschild formula, at that time, 1919, this result was not known because light "had" an unquestionable rectilinear path, and a surprised English astrophysic, Sir **Arthur Stanley Eddington**, who promoted to observe the total sun eclipse of 29 May 1919 to test Einstein's ideas about bending light, launched Einstein to celebrity.

Obviously, in our opinion Newtonian prediction of bending light is without doubt conceptually erroneous, because in its general analysis it does not consider the relativistic mass dependence on its velocity and on the speed of light. Thus, its results are applicable only for constant masses. For instance, for analyzing bending of light in which photon always, in our opinion, is moving at speed c, has an initial momentum p_0 before being attracted by the massive body, and of course an initial mass m_0, it varies with the nearness to the massive body to a generic momentum p and mass m. Due to Newtonian analysis does not consider these variable aspects of photon's movement, of course, it gives very approximated but erroneous results.

We want to emphasize that a common aspect with Newtonian approach is that mass variation is not considered for obtaining Schwarzschild geometry (?). Nevertheless, modern tests of Relativity had accounted confirmation for less than a ten percent, of Schwarzschild prediction, but criticisms like the previous ones, and others, have given as result approaches like ours identified as Vectorial Relativity. In sum, what is important is, always, conceptually speaking, the true, its correctness in its most clear presentation, no matter if gain in accuracy is not so relevant, because approximations are good enough.

As it can be observed, I need to declare that we don't say that GTR is wrong. We only call the attention on those "solutions" of Einstein's Field Equations where basic principles are not respected.

In order to be balanced in this criticism, let's put other examples of metrics with the same commented problem:

Reissner-Nordström metric, which gives a generalization of Schwarzschild solution to EFE for studying black holes, where the electric charge, Q, of the black hole is considered:

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right).c^2.dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2.d\theta^2 + r^2.\sin^2\theta.d\phi^2 \tag{6}$$

Nevertheless, there are some solutions to EFE without this factor multiplying the term c^2t^2, as it is the Friedman-Lemaître-Robertson-Walker metric (FLRW) which rules the evolution of the universe:

$$ds^2 = -c^2.dt^2 + R^2(t).\left(\frac{dr^2}{1 - k.r^2} + r^2.d\theta^2 + r^2.\sin^2\theta.d\phi^2\right) \tag{7}$$

where r, θ, ϕ are the polar coordinates, and R(t) the scale factor (positive), and $k = +1, 0, -1$, depending upon the geometry of the universe.

For radial light photons where $d\theta = d\phi = 0$, and null geodesics where $ds = 0$, equation (15) reduces to:

$$0 = -c^2.dt^2 + R^2(t).\left(\frac{dr^2}{1 - k.r^2}\right) \quad \Rightarrow \quad c^2dt^2 = dr^2.\left(\frac{R^2(t)}{1 - k.r^2}\right) \Rightarrow \frac{dr}{dt} = \frac{\pm c}{\sqrt{\dfrac{R^2(t)}{1 - k.r^2}}}$$

Also in this solution, where the term $c^2.dt^2$ doesn't contain any factor, it doesn't ensure the constancy of photon's speed in any situation because dr^2 is multiplied by the factor $\left(\frac{R^2(t)}{1-k.r^2}\right)$, thus holding again the contradictions observed for the Schwarzschild solution to Einstein's Field Equations, presented in the first part of this section.

III. CONCLUSION

It can be observed that if either terms $c^2.dt^2$ and dr^2 within any metrics of solutions to EFE are preserved "clean" or containing multiplying factors that cancel out when obtain the radial speed of photon, in a similar way as that followed here, it would guarantee the preservation of the speed of light as a constant within the results given by solutions to EFE. In our opinion, factor $\left(1-\frac{r_S}{r}\right)$ multiplying $c^2.dt^2$, and factor $\left(1-\frac{r_S}{r}\right)^{-1}$ multiplying dr^2 are the source of the inconsistent result displayed in equation (2). Also, the same situation is seen in the Reissner-Nordström metric, equation (6). A different equation presenting the same inconsistencies is the Friedman-Lemaître-Robertson-Walker metric in equation (7).

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