

Review

Quantum Mechanics in Vectorial Relativity

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ABSTRACT: In previous work it was shown that assumptions, $y' = y$ and $z' = z$, within Lorentz Transformations (LT) were needless, and therefore groundless. Achieved development of LT without assumptions, brought about a unique relativistic mass definition, $m = \frac{M_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}}$. Also, in a subsequent

work, based on this definition of mass, a new and general expression of relativistic Energy was devised, $E = 2.M_0 - m.(c^2 - 2.v^2)$, valid for particles with null or non-null masses at rest. Remember that it reduces to Einstein's equation, $E = m.c^2$ for particles with null mass at rest (i.e. photons) and to Newtonian Energy, $E = \frac{1}{2}m.v^2$, for particles with non-null mass at rest and moving at low speeds $v \ll c$. Nevertheless, although $E = m.c^2$ works as a very good approximation in energy calculations for bodies with non-null mass at rest, it was demonstrated in such work that at speeds more than two thirds that of light, erroneous outputs are obtained. In the current review the modified Schrödinger Equation, obtained from the new relativistic energy equation, is revisited.

KEYWORDS: Relativistic Energy, Schrödinger Equation (Modified), Vectorial Quantum Mechanics .

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I. INTRODUCTION

The beginnings of twentieth century was very productive for science and specially for Quantum Physics. In 1901 Max Planck established completely his work on black body. But for this work he already accounted since 1889 with the relationship between the luminous energy E and frequency

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of emitted light by the black body ν through his famous constant, $E = h\nu$. Later, in 1905 Albert Einstein published his seminal paper on the Special Theory of Relativity, in where he indirectly presented his celebrated equation relating the energy of a moving particle with its mass and speed of light, $E = mc^2$. In this same subject Einstein worked on the photoelectric effect and the light quanta, which was checked as correct by Millikan in 1916 and served for assign to Einstein the Nobel price in 1921. In 1923-1924 Louis-Victor De Broglie presented his eminent hypothesis about wave-particle duality as an inner characteristic of matter with application through the wave equation, which, after many controls and experiments was attained for a Nobel Prize in 1929. But two years later, in 1925-1926, in order to complete that work initiated by Planck, Nobel Prize in 1918, for establishing the foundations of Quantum Mechanics, continued by Einstein and Louis-Victor De Broglie, the Austrian physicist Erwin Schrödinger established his prominent and controversial equation by expanding meaning of matter to a probabilistic particle-wave character. In 1929, British mathematician Paul Adrien Maurice Dirac made an effort to mathematically relate the new visions of physics: Quantum Mechanics and Relativistic theory. Dirac shared with Schrödinger the Nobel Prize in 1933 by their contributions to Quantum Mechanics. In 2006, recently Franco's correction to relativistic mass [1] and to relativistic energy [2] definitions definitely will enrich this process of searching.

This Review is presented in the following order: In Section II, Quantum Theory of light is introduced. In Section III is shown a possible way for obtaining De Broglie's hypothesis starting from the Einstein's Energy equation for photons. In Section IV, Schrödinger Equation is discussed. In Section V a modified Schrödinger Equation is presented based upon the new definition of relativistic Energy, and in Section VI are revisited some applications of Relativistic Quantum Mechanics under Vectorial Relativity.

II. QUANTUM THEORY OF LIGHT

A fundamental concept in this theory started with the discovering in 1889 of the Planck's Constant with which luminous energy, E , appears as packets of light, or photons or light quanta, each with a value that can be calculated by Planck's relationship:

$$E = h\gamma \Leftrightarrow \gamma = \frac{c}{\lambda}, \text{ for a photon with frequency } \gamma, \text{ wavelength } \lambda, \text{ and speed } c.$$

Analysis done by Planck has been confirmed by experimental tests. Another contribution was done by Einstein in 1905, where energy of Photon can be calculated also by using his famous equation, which relates the mass of the photon, as it were a particle, and the speed of light, $E = mc^2$, and a relationship between photon's linear momentum, p , and photon's wavelength, through Planck's constant is obtained:

$$E = mc^2 = m.c.c = p.c = p.\lambda.\gamma \Rightarrow E = p.\lambda.\gamma = h.\gamma$$

From here, the product of the light wavelength times linear momentum is constant and equal to the Planck's constant:

$$h = p \cdot \lambda \quad (1)$$

Einstein's energy equation for light, considers photon as matter or particle, because they assign a mass to it, and Planck's energy equation for light ratifies light as a wave. Thus, light has two intrinsic features: a particle character but also a wave character. Moreover, from Planck's result it is noticeable that each photon has a packetized energy, or quantized energy, in the sense that total energy conveyed by a stream of photons, at a determined frequency, equals the discrete sum of all these elemental packets, and furthermore, there are no fractional values of these elemental packets.

Discrete values or quantum values of some magnitudes is apparently a natural characteristic. For example, Electric Charge is another magnitude with this characteristic.

These facts, may be directed De Broglie to make is proposition of the wave-particle duality as a general characteristic of matter, which we have started discussing in this section.

III. DE BROGLIE'S WAVE-PARTICLE DUALITY (1923-1924)

De Broglie established in 1924 a wavelength associated to a particle whose mass m moves at speed v , that should satisfy same relationship as that for photon, although he also proposed this in his PHD thesis in 1923:

$$\lambda = \frac{h}{p} = \frac{h}{m \cdot v} \quad (2)$$

Let's analyze this relationship. At that moment, De Broglie knew about energy's Planck's relationship, $E = h \cdot \gamma$, and Einstein's equation, $E = m \cdot c^2 = p \cdot c$. From this he readily deduced that:

$$pc = h \cdot \gamma \Rightarrow h = p \cdot \frac{c}{\gamma} = p \cdot \lambda \Rightarrow h = p \cdot \lambda \Rightarrow \lambda = \frac{h}{p} \quad (3)$$

So, De Broglie decided, in authors' opinion, to expose the hypothesis that, in general, any particle in motion has an associated wave whose wavelength has a value of: $\lambda = \frac{h}{m \cdot v}$.

Theoretical results obtained by Bohr in his analysis of the hydrogen atom and those observed in Quantum Mechanics gave validity to this relationship. Although the relationship comes up from an expression that is only true for photons, as it is demonstrated in [2], it has been accepted because the experimental facts support it and we will agree to its effectiveness in this work.

Such relationship is of transcendental importance in this work as it was in Quantum Theory. Thus, introducing it as well as the well-known expression between velocity of a wave and frequency and wavelength, $v = \lambda \cdot \gamma$ can be possible to generalize the application of equations to speeds less than the speed of light c .

IV. SCHRÖDINGER'S EQUATION (1925-1926)

In order to develop and enlarge the meaning of De Broglie's hypothesis, we believe that Schrödinger primary idea was to relate particle's momentum with the wave number k , appearing in the wave equation of a string, air, or electromagnetic waves. In all these cases such equation is the same. The referred wave equation in its one-dimensional presentation follows:

$$\frac{d^2\xi}{dx^2} + k^2 \cdot \xi = 0, \quad \text{where } k = \frac{2\pi}{\lambda} \quad \text{is the wave number for any type of wave.}$$

The well known solutions of this equation are: $\xi = e^{jkx}$ and $\xi = e^{-jkx}$.

Thus, by using $p = \frac{h}{\lambda}$ and doing $p = \frac{2\pi}{\lambda} \frac{h}{2\pi} = \hbar \cdot k$, where $\hbar = \frac{h}{2\pi}$, Schrödinger construct this one-dimensional equation not depending on time:

$$\frac{d^2\xi}{dx^2} + k^2 \cdot \xi = 0 \Rightarrow \frac{d^2\xi}{dx^2} + \frac{p^2}{\hbar^2} \cdot \xi = 0 \quad (4)$$

Conversely, in classic physics, total energy E can be constituted by the Kinetic energy $K = \frac{1}{2}m \cdot v^2$ plus the potential energy E_p , i.e:

$$E = \frac{1}{2}m \cdot v^2 + E_p = \frac{1}{2} \frac{p^2}{m} + E_p \quad (5)$$

From here, the expression of momentum follows:

$$p^2 = 2 \cdot m \cdot (E - E_p) \quad (6)$$

Substituting this last expression and simplifying, we arrive at the first Schrödinger one-dimensional equation not depending on time:

$$-\frac{\hbar^2}{2 \cdot m} \cdot \frac{d^2\xi}{dx^2} + E_p \cdot \xi = E \quad (7)$$

However, a relativistic physicist would say that there exists error in the previous equation because the used kinetic energy expression is not exact from the relativistic point of view. But a quantum physicist would reply that the reason for using the classical equation of kinetic energy is due to Einstein's relativistic expression of kinetic energy does not depend directly and explicitly on particle's velocity, which makes difficult the presentation Quantum Mechanics based on relativistic relationships. Despite of all these arguments, P. A. M. Dirac developed Schrödinger's equation based on Einstein's relativity, with its solutions (!). But very far from thinking that all problems are already finished, Franco's corrections to Einstein's relativistic expressions of energy, linear

momentum and mass [1] [2] introduced new conceptual constraints. In next section, our point of view on this problem is presented.

V. SCRÖDINGER'S EQUATION UNDER VECTORIAL RELATIVITY

As it should be expected, Franco's new definition of relativistic energy entails interesting consequences for quantum mechanics (QM). As we previously said, the reason to use the classical equation of energy in quantum theory, in addition to give good experimental results, is that Einstein's relativistic expression of kinetic energy does not depend directly and explicitly on the velocity of the particle as it can be encountered in Newtonian Mechanics (which allows the direct relationship between the classic Hamiltonian with the Quantum operator), it introduces mathematical complications, difficult to handle by QM physicists, although these problems, were finally solved by P. A. M. Dirac [3].

For instance, under vectorial relativity kinetic energy of a particle starting its movement from rest, *depends directly and explicitly on the velocity of the particle*:

$$K = 2.m.v^2 - m.c^2 + M_0.c^2 = 2.\frac{p^2}{m} - c^2.(m - M_0) \quad (8)$$

Forming total energy E , by including kinetic, **internal** and potential energy, for ensuring that E preserves constant in any case (including those cases where part of the internal energy converts to any other kind of energy, or there are energy-mass interchanges), and acquiring the expression of momentum, we have:

$$E = K + M_0.c^2 + E_p = 2.m.v^2 - m.c^2 + 2.M_0.c^2 + E_p = 2.\frac{p^2}{m} - c^2.(m - 2.M_0) + E_p \quad (9)$$

This result recalls us the "classical" Hamiltonian. By using our relativistic expression in (10), for the case of three dimensions in the same way as we used the Classical Hamiltonian, we can re-define the Relativistic Hamiltonian as:

$$H_{relativistic} = 2.\frac{\mathbf{p}^2}{m} - c^2.(m - 2.M_0) + E_p(\mathbf{r}) \quad (10)$$

From equation (9) we obtain a suitable expression of the linear momentum:

$$p^2 = \frac{E - E_p + c^2.(m - 2.M_0)}{2}.m \quad (11)$$

Let's do the following exercise: with the relationship (11) we will try to obtain our version of the Schrödinger Equation. For such task we are going to use the analogy between the Wave and Schrödinger Equations. We need to say that we know that this way of introducing Schrödinger

Equation is not so like-minded, but, in author's opinion, it is very illustrative and may be this was the way Schrödinger used in 1926 to obtain his equation, trying to extend the wave-particle interpretation put forward in 1924 by De Broglie. This development, author insists, is very illustrative and simple to understand. By the way, it is noteworthy that Schrödinger was opposed at that time, 1930, to the probabilistic interpretation given by Bohr to his equation [4]. Well, the intention here is to obtain direct results through simple analogies with quantum mechanics for the sake of synthesizing the applicability of deduced expressions, but at no moment the author wants to coin that this is a meticulous form to derive the Schrödinger's equation. This showing is only for quickly promoting the obtaining of new relativistic operators in Quantum Mechanics, given the direct and explicit dependency on velocity of Franco's energy expression.

Let γ be the frequency of the wave, λ be its wavelength, k be the wave number, where $k = \frac{2.\pi}{\lambda}$ and h be the Planck constant, for $\hbar = \frac{h}{2.\pi}$.

By associating a one-dimensional periodical wave, given by $\xi = \xi_0.\sin k(x - v.t)$ to a particle that travels at a velocity v along the X-axis, the wave equation for this case becomes:

$$\frac{d^2 \xi}{dx^2} + k^2 .\xi = \frac{d^2 \xi}{dx^2} + \frac{p^2}{\hbar^2} .\xi = 0, \quad (12)$$

The velocity of the particle v , which is the same of its associated wave with wavelength λ and frequency γ , holds $v = \gamma.\lambda$. Let's take the same De Broglie's assumption that the product of the wavelength λ times the linear momentum p equals the Planck constant, in order to obtain the relationship between the linear momentum p and the wave number k :

$$h = p.\lambda \Rightarrow p = h.\frac{1}{\lambda} = \frac{h}{2.\pi} \frac{2.\pi}{\lambda} = \hbar.k,$$

In contrast, in this analysis of waves associated to particles with non-null mass at rest we have checked and concluded that the energy of a moving particle with non-null rest mass can not be considered equal to the Planck constant times its wavelength, $E \neq h.\gamma$, as it is for photons. Thus, by introducing the new momentum expression of equation (11) into equation (12), we come up with the Schrödinger spatial equation in one dimension:

$$-\frac{2.\hbar^2}{m} \cdot \frac{d^2 \xi}{dx^2} + [E_p - c^2.(m - 2.M_0)]\xi = E.\xi \quad (13)$$

To arrive at the general expression of Schrödinger equation, we are following a similar procedure to that presented by Alonso & Finn in [5]. Thus, we will be concerned in looking for a wave function that depends on the space and the time, which after deriving it with respect to the time and space we would have expected to obtain the spatial equation (13). The General Equation encountered by Schrödinger was:

$$-\frac{2\hbar^2}{m} \cdot \frac{\partial^2 \psi}{\partial x^2} + [E_p - c^2 \cdot (m - 2M_0)]\psi = j\hbar \cdot \frac{\partial \psi}{\partial t} \quad (14)$$

A function that fulfills those requirements is a product of two separate variables:

$$\psi(w,t) = \xi(x) \cdot e^{-\frac{j \cdot E \cdot t}{\hbar}}; \text{ where, } \frac{\partial \psi}{\partial t} = -\frac{j \cdot E}{\hbar} \cdot \xi(x) \cdot e^{-\frac{j \cdot E \cdot t}{\hbar}}; \text{ and } \frac{\partial^2 \psi}{\partial x^2} = \frac{\partial^2 \xi}{\partial x^2} \cdot e^{-\frac{j \cdot E \cdot t}{\hbar}} \quad (15)$$

By substituting these results in (15) we finally obtain, as expected, the Schrödinger spatial equation (14). Allowing the correlation between relativistic Hamiltonian, in the new conception, and that of quantum mechanics, the following associated operators take place:

$\mathbf{r} \Rightarrow \mathbf{r}$	(Position)
$\mathbf{p} \Rightarrow -j\hbar \cdot \nabla$	(Linear Momentum)
$\mathbf{rxp} \Rightarrow j\hbar \cdot \mathbf{rx} \nabla$	(Angular Momentum)
$2 \cdot \frac{\mathbf{p}^2}{m} - c^2 \cdot (m - M_0) \Rightarrow 2 \cdot \frac{\hbar^2}{m} \nabla^2 - c^2 \cdot (m - M_0)$	(Kinetic Energy)
$2 \cdot \frac{\mathbf{p}^2}{m} - c^2 \cdot (m - 2M_0) \Rightarrow 2 \cdot \frac{\hbar^2}{m} \nabla^2 - c^2 \cdot (m - 2M_0)$	(Total Energy)

As it can be observed from these analogies, only the known quantum operators of energy are slightly modified.

VI. CONCLUSION

So, matter shows a double behavior: as particle and as wave. We have observed that Vectorial Relativity, as a correct view of the particle character of matter, introduced in the basic equation of wave leads to the complete interpretation of matter. The conceptual bridge that unifies both characters, imposing quantization restrictions, is the general and well known constant relationship, for any particle, between its linear momentum and its coupled wavelength, $h = p \cdot \lambda$, which is possible thanks the contribution of Planck, Einstein and De Broglie. With such concept Quantum and Relativistic views of matter are unified through the Schrödinger Equation.

Alternatively, the introduction of the relativistic concepts into the Schrödinger Equation was achieved in a simple way, thanks the direct dependence on velocity of vectorial energy expressions. We believe that vectorial relativity has set a simple and direct bridge between Relativistic Theory and Quantum Mechanics, signifying a contribution to the unification of physics, conceptually speaking.

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