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Letter

Bending of Light in Vectorial Relativity

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ABSTRACT: It is discussed bending of light under the optics of Vectorial Relativity. An approximation formula is presented for calculating the total deflection of a photon that suddenly falls in a strong gravitational field.

KEYWORDS: Universal Gravitation Law, Bending of Light, Vectorial Relativity.

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I INTRODUCTION

Throughout the article we will assume that Relativity is the proper description of nature in the sense of mass variation, though we don't say anything about what kind of Relativity in order to allow the possibility of application of any theory. Although, as we hope, students and researchers who are not necessarily experts in relativity will use this article, we have developed the discussion with the simplest mathematical tools required for building models of relativistic objects. Given the potential for future applications of this formalism, we have opted to base much of our description on our previous work on a similar topic [1].

II BENDING OF LIGHT

In previous work, in this same issue [2] (*Precession in Vectorial Relativity*) we obtained that the value of angle, starting from perihelion, is given by:

$$\theta = \int_{u_0}^{u} \frac{-du}{\sqrt{.(u_0 - h.f(u))^2 - (u - h.f(u))^2}} \qquad (1)$$

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For $u = \frac{1}{r}$; $h = \frac{G.M.p_0^2}{K^2}$; $K = \omega.r^2.p = \omega_0.r_0^2.p_0$; $f\left(\frac{1}{r}\right) = \sqrt{\left(\frac{G.M}{c^2}.\left(\frac{1}{r_0} - \frac{1}{r}\right)\right)^2 + 1} - \frac{G.M}{c^2}.\left(\frac{1}{r_0} - \frac{1}{r}\right)$

In where, the integral in (1), can be approximately calculated by the following expression [2]:

$$\theta = \sum_{i=1}^{N} \left\{ arc\cos\left(\frac{\frac{1}{r_i} - h.f(r_{i-1,i})}{\frac{1}{r_0} - h.f(r_{i-1,i})} \right) - arc\cos\left(\frac{\frac{1}{r_{i-1}} - h.f(r_{i-1,i})}{\frac{1}{r_0} - h.f(r_{i-1,i})} \right) \right\} \quad (2)$$

For $r_{i-1} = \frac{r_0}{1-(i-1)\frac{1-r_0/r_a}{N}}$, $r_i = \frac{r_0}{1-i\frac{1-r_0/r_a}{N}}$; $r_{i-1,i} = \frac{r_0}{1-(i-1/2)\frac{1-r_0/r_a}{N}}$

This is based on the following equality:

$$\theta = \int_{u_0}^{u} \frac{-du}{\sqrt{.(u_0 - h.f(a))^2 - (u - h.f(a))^2}} = \arccos\left[\frac{u - h.f(a)}{u_0 - h.f(a)}\right]_{u_0}^{u}, \quad \text{For } h,\ f(a) \text{, constant.} \quad (3)$$

A. Past history of photon, before its approximation to the massive body.

Let's start supposing that a free photon travels in the space with a momentum p_∞ at a rectilinear speed c without experiencing any kind of disturbance, and suddenly it falls in the gravitational field of attraction of a massive body. Then, its path starts leaving of being rectilinear to convert into curvilinear. So, photon approaches to the massive body until reaching a minimum distance, or minimum radius r_0. If eccentricity, $e = \frac{r_0}{r_0^{'}} - 1$, is less that unity which gives something like an ellipse, it means that for our version of Schwarzschild Radius, $r_0^{'} = \frac{G.M}{c^2}$, this implies $r_0 < 2.r_0^{'}$. For $r_0 \approx 2.r_0^{'}$ we are in front of a parabola and for $r_0 > 2.r_0^{'}$a, orbit is a hyperbola. But in this case, we are in a different situation to that previously presented in the analysis done for Photon's Precession in [1]. Although obtaining same equations, when photon approaches to massive body its radius decreases, of course its inverse $u = \frac{1}{r}$ increases, then mathematically the function that holds for this case is $\arcsin$. This means that solution presented in (3), applied to any parabolic or hyperbolic approaching motion, displays as:

$$\varphi = \int_{u_0}^{u} \frac{-du}{\sqrt{.(u_0 - h.f(a))^2 - (u - h.f(a))^2}} = arcsin\left[\frac{u - h.f(a)}{u_0 - h.f(a)}\right]_{u_0}^{u}, \quad \text{For } h,\ f(a) \text{, constant.} \quad (4)$$

As we can observe, we have used equal intervals $\Delta = \dfrac{\dfrac{G.M}{c^2}.\left(\dfrac{1}{r_0} - \dfrac{1}{r_a}\right)}{N}$ and obtained the corresponding values taken by radius at the extremes of integration $r_{i-1} = \dfrac{r_0}{1-(i-1)\dfrac{1-r_0/r_a}{N}}$ and $r_i = \dfrac{r_0}{1-i\dfrac{1-r_0/r_a}{N}}$ and that at the middle of the interval, $r_{i-1,i} = \dfrac{r_0}{1-(i-1/2)\dfrac{1-r_0/r_a}{N}}$ for evaluating $f(r_{i-1,i})$ (the number of intervals need to be a great number N to ensure precision). This easy way of estimation is also suitable for the calculation of angle in the case of parabolic motion of photon, because the total length between extremes of integration, the basis of a triangle-rectangle, from where intervals were chosen, is a finite quantity, and thus the intervals are also finite. In fact, the expression of the basis of the triangle-rectangle is: $\dfrac{G.M}{c^2}.\left(\dfrac{1}{r_0} - \dfrac{1}{r_a}\right) \le 1$. Its maximum value, for $r \to \infty$, in a parabolic or hyperbolic motion, will be finite and positive, $\dfrac{G.M}{c^2}.\left(\dfrac{1}{r_0} - \dfrac{1}{\infty}\right) = \dfrac{G.M}{c^2.r_0}$; Thus, for $r = r_0$, It becomes $\dfrac{G.M}{c^2}.\left(\dfrac{1}{r_0} - \dfrac{1}{r_0}\right) = 0$; and for $0 < r < r_0$, its real value could be big but finite. It necessary to say that only in the case of Spherical Black Holes, in where the radius from its center to its surface, r_s could be smaller than our expression of the Schwarzschild radius $\dfrac{G.M}{c^2}$, which is half of the radius coined by Schwarzschild, $\dfrac{2.G.M}{c^2}$. In "elliptical" motion of photons, the major axis, that given by the summation of radius at perihelion (minimum radius) plus radius at aphelion, is constant and equal to: $r_a + r_0 = \dfrac{2.G.M}{c^2}$, namely, what is gained by one of these two radiuses is lost by the other. In sum, the approximate bending angle previous to the nearest photon approaching to massive body is:

$$\varphi_{i-1,i} = \int_{r_{i-1}}^{r_i} \frac{-du}{\sqrt{.(u_i - h.f(u))^2 - (u_{i-1} - h.f(u))^2}} = arc\,sin\left(\frac{\dfrac{1}{r_i} - h.f(r_{i-1,i})}{\dfrac{1}{r_0} - h.f(r_{i-1,i})}\right) - arc\,sin\left(\frac{\dfrac{1}{r_{i-1}} - h.f(r_{i-1,i})}{\dfrac{1}{r_0} - h.f(r_{i-1,i})}\right) \quad (5)$$

$$\varphi_1 = \int_{r_0}^{r_a} \frac{-du}{\sqrt{.(u_0 - h.f(u))^2 - (u - h.f(u))^2}} = \sum_{i=1}^{N}\left\{arc\,sin\left(\frac{\dfrac{1}{r_i} - h.f(r_{i-1,i})}{\dfrac{1}{r_0} - h.f(r_{i-1,i})}\right) - arc\,sin\left(\frac{\dfrac{1}{r_{i-1}} - h.f(r_{i-1,i})}{\dfrac{1}{r_0} - h.f(r_{i-1,i})}\right)\right\} \quad (6)$$

Lets suppose that mass of the massive body is null, $h = \frac{G.M.p_0^2}{K^2} = 0$, and the nearest point occurs at $r = R_0, \Rightarrow u = U_0$, where radius R_0 is greater than previously referred minimum radius r_0. Thus, radius of photon coming from infinite has the following expression:

$$\varphi = arcsin\left[\frac{\frac{1}{r} - h}{\frac{1}{R_0} - h}\right]_{\infty}^{R_0} = arcsin\left[\frac{\frac{1}{R_0}}{\frac{1}{R_0}}\right] - arcsin\left[\frac{\frac{1}{\infty}}{\frac{1}{R_0}}\right] = \frac{\pi}{2} \quad (7)$$

On the other hand, because in this case the evaluation of angle at infinite is null, we can put that for any radius between R_0 and ∞, for $M = 0$:

$$\varphi = \arcsin\left[\frac{\frac{1}{r} - h}{\frac{1}{R_0} - h}\right]_{\infty}^{R_0} = \arcsin\left[\frac{\frac{1}{r}}{\frac{1}{R_0}}\right] - \arcsin\left[\frac{\frac{1}{\infty}}{\frac{1}{R_0}}\right] \Rightarrow \frac{\frac{1}{r}}{\frac{1}{R_0}} = sin\varphi \Rightarrow r = R_0.\csc\varphi. \quad (8)$$

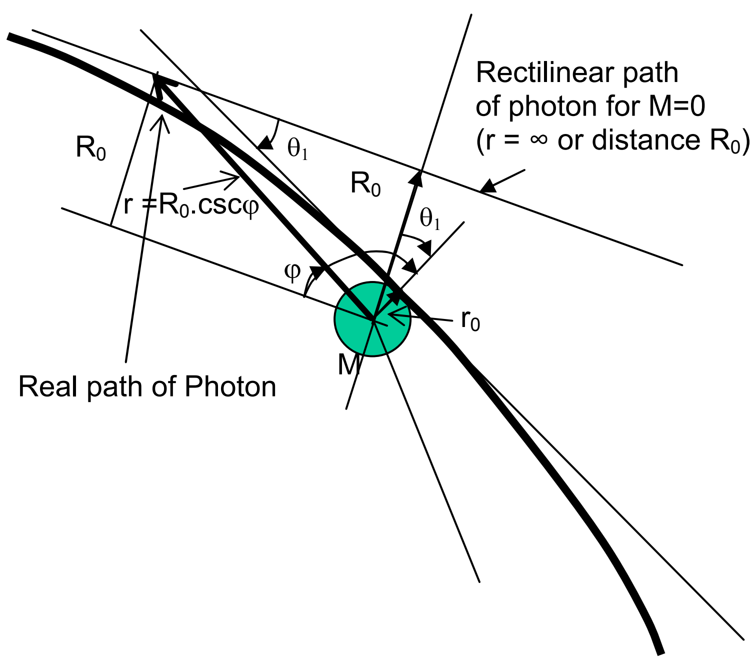

Fig. 1

The two bents of photon's path, before and after maximum approaching, are equal and symmetric. In fact, suppose and think of constant mass, from $r = \infty, \Rightarrow u = 0$, until $r = r_0, \Rightarrow u = u_0$:

$$\varphi = \int_{u_0}^{u} \frac{-du}{\sqrt{.(u_0 - h.f(a))^2 - (u - h.f(a))^2}} = \arcsin\left[\frac{u - h.f(a)}{u_0 - h.f(a)}\right]_{\infty}^{u_0} = \arcsin\left[\frac{u_0 - h.f(a)}{u_0 - h.f(a)}\right] - \arcsin\left[\frac{-h.f(a)}{u_0 - h.f(a)}\right]$$
$$= \frac{\pi}{2} - \arcsin\left[\frac{-h.f(a)}{u_0 - h.f(a)}\right] = \frac{\pi}{2} + \arcsin\left[\frac{h.f(a)}{u_0 - h.f(a)}\right] \quad (9)$$

As it can be seen from the graph, when in the real path generic radius reaches $r = r_0, \Rightarrow u = u_0$, partial angle deflected by photon path is

$$\varphi - \frac{\pi}{2} = \frac{\pi}{2} + \arcsin\left[\frac{-h.f(a)}{u_0 - h.f(a)}\right] - \frac{\pi}{2} = \arcsin\left[\frac{-h.f(a)}{u_0 - h.f(a)}\right] = \theta_1 \tag{10}$$

In the next sub-section we will obtain the other part of the deflection.

B. Present history of photon, after its approximation to the massive body.

The same last expression will be obtained by calculating the bending of light after the minimum radius $r = r_0$ and reaching again $r = \infty, \Rightarrow u = 0$, In this situation function $\arccos$ holds:

$$\phi = \int_{u_0}^{u} \frac{-du}{\sqrt{.(u_0 - h.f(a))^2 - (u - h.f(a)^2}} = \arccos\left[\frac{u - h.f(a)}{u_0 - h.f(a)}\right]_{u_0}^{\infty} =$$
$$= \arccos\left[\frac{-h.f(a)}{u_0 - h.f(a)}\right] - \arccos\left[\frac{u_0 - h.f(a)}{u_0 - h.f(a)}\right] = -\arccos\left[\frac{-h.f(a)}{u_0 - h.f(a)}\right] - 0 = \arccos\left[\frac{-h.f(a)}{u_0 - h.f(a)}\right] \tag{11}$$

The last expression comes from the fact that the function $\cos(-\alpha) = \cos\alpha$. And as it can be seen from next graph, when in the real path generic radius reaches $r = \infty, \Rightarrow u = 0$, real angle deflected by photon path is:

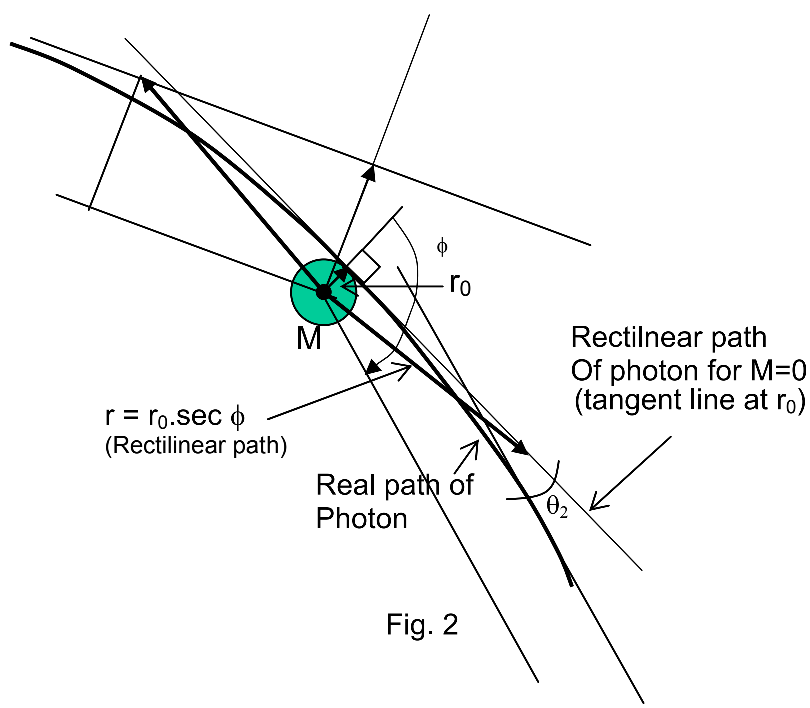

Fig. 2

$$\phi-\frac{\pi}{2}=\arccos\left[\frac{-h.f(a)}{u_0-h.f(a)}\right]-\frac{\pi}{2}=\arcsin\left[\frac{-h.f(a)}{u_0-h.f(a)}\right]=\theta_2 \quad (12)$$

Given that $\cos\left(\alpha-\frac{\pi}{2}\right)=\sin\alpha$. Thus, in this way we have shown that $\theta_1=\theta_2$

The total bending will be given by:

$$\theta=\theta_1+\theta_2=2.\arcsin\left[\frac{-h.f(a)}{u_0-h.f(a)}\right], \quad \text{For } h,\ f(a)\text{, constant.} \quad (13)$$

In this way we can assure that an approximate calculation of total bending will be then:

$$\theta=\int_{r_0}^{r_a}\frac{-du}{\sqrt{(u_0-h.f(u))^2-(u-h.f(u))^2}}=2.\sum_{i=1}^{N}\left\{arc\sin\left(\frac{\frac{1}{r_i}-h.f(r_{i-1,i})}{\frac{1}{r_0}-h.f(r_{i-1,i})}\right)-arc\sin\left(\frac{\frac{1}{r_{i-1}}-h.f(r_{i-1,i})}{\frac{1}{r_0}-h.f(r_{i-1,i})}\right)\right\} \quad (14)$$

For: $$r_{i-1}=\frac{r_0}{1-(i-1)\frac{1-r_0/r_a}{N}}\rightarrow r_i=\frac{r_0}{1-i\frac{1-r_0/r_a}{N}};\quad r_{i-1,i}=\frac{r_0}{1-(i-1/2)\frac{1-r_0/r_a}{N}} \quad (15)$$

With these tools in hand it can be obtained an approximated value of total angle of light deflection.

III CONCLUSIONS

As in previous definitions of mass or energy, again we are in the same situation: It is necessary to check the experimental validation of this approach. For example, until now we have not found any relation either between gravitation and time or between speed of light and gravitation as that of a speed of a radar signal going radial to gravitational force, which so it behaves according to Einstein's General Theory of Relativity. Nevertheless, accuracy of concepts expressed here, will probably require further research and experimentation in order to establish the correctness of our work. These tasks probably could be possible to achieve by next years (see second article of section News, in this issue and also in previous one).

REFERENCES

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