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Review

Energy in Vectorial Relativity

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ABSTRACT: In previous Franco's work it was made clear that assumptions, $y' = y$ and $z' = z$, within Lorentz Transformations were needless, and because of such assumptions, Lorentz Transformations (LT) depend on the body's spatial orientation, i.e. the well-known transverse and longitudinal transformations of magnitudes, characterized by different scaling factors. On the contrary, the development of LT without assumptions, brought about new transformations that do not depend on spatial orientation, and a unique mass definition was devised, $m = \frac{M_0}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}}$. As it is known, Einstein arrived at two definitions: transverse mass $m_T = \frac{M_0}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}$ and longitudinal mass $m_L = \frac{M_0}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}}$, but without any explanation Einstein coined the former and discarded the latter (!). Based on this unique definition of mass new general expressions of Energy and Momentum were derived, which guided to conclude that Einstein's equation $E = m.c^2$ is only valid for particles with null mass at rest (i.e. photons). However, it is noticeable that $E = m.c^2$ works as a very good approximation in energy calculations for bodies with non-null mass at rest, at speeds less than two thirds that of light.

KEYWORDS: Special Relativity, Relativistic Mass, Relativistic Energy and Relativistic Momentum.

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I. INTRODUCTION

The concept of variation of a mass m with its velocity v, through its rest mass M_0 and the universal constant speed of light c, in rectilinear motion, given by $m = \frac{M_0}{\sqrt{1-\frac{v^2}{c^2}}}$, was originally established by Einstein in the section §10, of his seminal paper about relativity on June 30th, 1905 [1]. Doubtlessly, the mass dependence of a body on its velocity was one of the major outcomes of the Special Theory of Relativity. Additionally, in the same section §10, Einstein *indirectly* sets up (with another notation) his referred famous equation, through the derivation of the kinetic energy of the electron, $K = m.c^2 - M_0.c^2$. In where K denoted the Kinetic energy of the electron and $m = \gamma.M_0$ its relativistic mass, for $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, moving at velocity v. Later, on September 27th 1905, in a second paper he formally presented this result by establishing: "*the mass of a body is a measure of its energy contents*", referring this comment to the previous definition of energy [2]. However, *"The first record of the relationship of mass and energy explicitly in the form $E = m.c^2$ was written by Einstein in a review of relativity in 1907",* After that, *"Relativistic mass came into common usage in the relativity text books of the early 1920s written by Pauli, Eddington and Born"* [3].

Some light criticisms about the Einstein's first derivation of relativistic mass in 1905 can be read in [3] [4] [5] [6] [7].

It is important to observe that Einstein did not derive his energy equation from the transverse mass definition as it is presented in the First Part of this work. In its place, he originally started from the assumption of equating the energy withdrawn from the electrostatic field, to the "energy of motion" of the electron. The inter-conversion mass-energy coined by Einstein in his equation, $E = m.c^2$, has been accepted and interpreted in different ways by scientists of any kind, practically since that year 1905 when it was officially presented to the scientific world in the German publication *Annalen der Physic*. On the other hand, there is an interesting work of Mendel Sachs in 1973 that calls the attention on the non-consideration of the change of nuclear configuration energies within the atom in a re-examination of such equation [8].

This work is presented in the following order: In Section II, is shown a simple way for obtaining the Einstein's equation of energy, $E = m.c^2$, starting from the Einstein's transverse mass definition (incorrect, according to [9]). In Section III, the unique definition of mass given in [9] was used instead and a new Energy Equation was obtained.

II. ENERGY DERIVATION FROM EINSTEIN'S MASS.

In the next paragraphs we will present a simple way of deriving the Einstein's energy definition from the transverse mass definition, following a similar procedure to that appeared in [10].

The classical Newton's second law, in its modern presentation, correctly establishes that the net Force $\mathbf{F}$ exerted on a mass m, equals the derivative relative to time of its Linear Momentum, $\mathbf{p}$ (the product of its mass m times its velocity $\mathbf{v}$), $\mathbf{F} = \frac{d(m.\mathbf{v})}{dt}$. By considering mass as variable, Newton's 2nd Law becomes:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d(m.\mathbf{v})}{dt} = m.\frac{d(\mathbf{v})}{dt} + \mathbf{v}.\frac{d(m)}{dt} \qquad (1)$$

Kinetic Energy, dK, or the work done by a force $\mathbf{F}$ that makes a mass m to have a displacement $d\mathbf{s}$ becomes, applying the vectorial identity, $d(\mathbf{A}.\mathbf{A}) = 2\mathbf{A}.d\mathbf{A} = d(A^2) = 2A.dA \Rightarrow \mathbf{A}.d\mathbf{A} \equiv A.dA$:

$$dK = \mathbf{F}.d\mathbf{s} = \frac{d(m.\mathbf{v})}{dt}.d\mathbf{s} = d(m.\mathbf{v}).\frac{d\mathbf{s}}{dt} = d(m.\mathbf{v}).\mathbf{v} = m.\mathbf{v}.d(\mathbf{v}) + \mathbf{v}.\mathbf{v}.dm = m.v.dv + v^2.dm \qquad (2)$$

By taking derivatives of Einstein's transverse mass definition, we obtain:

$$m = \frac{M_0}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \frac{dm}{dv} = \frac{M_0.c.v}{\left(c^2-v^2\right)^{\frac{3}{2}}} = \frac{M_0.c}{\left(c^2-v^2\right)^{\frac{1}{2}}}.\frac{v}{\left(c^2-v^2\right)} = \frac{M_0}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}.\frac{v}{\left(c^2-v^2\right)} = \frac{m.v}{\left(c^2-v^2\right)} \qquad (3)$$

From (3) and (2):

$$\frac{dm}{m} = \frac{v.dv}{\left(c^2-v^2\right)} \quad \Rightarrow \quad c^2.dm = v^2.dm + m.v.dv \quad \Rightarrow \quad c^2.dm = dK \qquad (4)$$

From equations (2) and (4), and integrating from <u>rest mass</u>, M_0, to a new generic state of mass m, Einstein's Kinetic energy expression is readily obtained:

$$K = \int (m.v.dv + v^2.dm) = \int_{M_0}^{m} c^2.dm \quad \Rightarrow K = m.c^2 - M_0.c^2 \qquad (5)$$

As a control of this, the expression (5) should reduce to the well-known Newton´s kinetics energy expression: $K = \frac{1}{2}.m.v^2$, for $v << c$. This fact is shown by expanding equation (5), i.e.:

$K = \frac{M_0.c^2}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}} - M_0.c^2$, as a binomial series of the type:

$$(1+x)^{-\frac{1}{2}} = 1 - \frac{1}{2}.x + \frac{1.3}{2.4}.x^2 - \frac{1.3.5}{2.4.6}.x^3 + ... \quad -1 < x \leq 1 \text{ for } x = \frac{v^2}{c^2}$$

$$\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = 1 + \frac{1}{2}.\left(\frac{v^2}{c^2}\right) + \frac{1.3}{2.4}.\left(\frac{v^2}{c^2}\right)^2 + \frac{1.3.5}{2.4.6}.\left(\frac{v^2}{c^2}\right)^3 + ... \quad \frac{v^2}{c^2} \leq 1$$

Substituting and discarding the terms divided by c^2, raised to any exponent:

$$K = M_0.\left[\left(c^2 + \frac{1}{2}.v^2 + ...\right) - c^2\right] \cong \frac{1}{2}.m.v^2 \qquad \text{for } m = \frac{M_0}{\sqrt{1 - \frac{v^2}{c^2}}} \cong M_0 \tag{6}$$

Remembering that Total Energy for a free particle, E, is the Kinetic energy K plus the internal energy $E_0 = M_0.c^2$, and by operating on equation (5) it follows:

$$E = .(m - M_0).c^2 + M_0.c^2 \quad \Rightarrow \quad E = m.c^2 \tag{7}$$

Thus, certainly Einstein's Energy equation in (7) is a direct consequence of the Einstein's transverse mass definition, and although "transverse energy" does not sound quite well, everything has remained until now "OK" with this definition. The "only" problem is that such mass definition, recently was determined as not correct in [9]. The simplicity of the previous derivation makes it beautiful. In author's opinion, Einstein was a victim of this illusion. It is worth to insist that Einstein's procedure used to arrive at his famous equation was based on the assumption of which the energy withdrawn from the electrostatic field develops into the energy of motion of an electron slowly accelerated. This is something completely different to the formal derivation used here *(also incorrect because it was done based upon an erroneous concept of mass)*.

A. Demonstration that something is wrong in Einstein's Definitions of mass and Energy.

In searching for a way that can allow us to demonstrate that Einstein's Energy equation is not a correct relationship, and convinced that if we start from a wrong definition we should arrive at a wrong relationship*,* we found in Einstein's Energy equation the following subtle mathematical disparity.

Let's try the subsequent mathematical test which will show a sort of inconsistency in this equation: For instance, if we asked ourselves, which are the maximum or minimum values that energy can obtain, with a velocity varying from $v = 0$ to $v = c$?. We can deduce from physical conditions that, obviously, these values are, $E_0 = M_0.c^2$ as a positive minimum, for $v = 0$, and an infinite value of energy for $v = c$. However, mathematically speaking, we are used to develop this task by taking the derivative of Energy respect to velocity v, and equating such result to zero. This procedure should lead us to obtain intersections of the derivative's path with the abscissa's axis, or velocity axis.

Firstly, the general definition of kinetic energy will be used in this search, because it is a proved formula in Physics. Thus, by taking the derivative of the kinetic energy relative to the velocity v and making it null and using obtained result in equation (2), we have a trivial solution:

$$\frac{dE}{dv} = \frac{d(K+E_0)}{dv} = \frac{dK}{dv} = 0; \quad m.v + v.v.\frac{dm}{dv} = 0 \Rightarrow v = 0 \Rightarrow K = 0 \Rightarrow \quad E = K + E_0 = E_0$$

$$\Rightarrow \quad m = -v.\frac{dm}{dv} \Rightarrow \quad \frac{dm}{m} = -\frac{dv}{v}$$

Now, allow the right hand side of this expression being equal to the right hand side of the first relationship in (4), $\frac{dm}{m} = \frac{v.dv}{c^2 - v^2}$, which, as we know, comes from Einstein's mass definition. This leads to:

$$-\frac{dv}{v} = \frac{v.dv}{c^2 - v^2} \qquad \Rightarrow \qquad -c^2 + v^2 = v^2 \qquad \Rightarrow \qquad c = 0 \qquad (!)$$

How to explain such "solution"? This is a contradictory and strange result, because speed of light cannot be zero, it is not a variable! A result like this means that there is something wrong somewhere. Let's see. We have used the general definition of Kinetic Energy, $dK = m.v.dv + v.v.dm$, which, as we know, is a correct definition in Physics and then problems do not come from this source. However, we also used in this derivation Einstein's definition of mass. Thus, because we don't have used any other physical definition, the only reason for obtaining such erroneous result must be that Einstein's is an incorrect mass definition (ratifying what was known from previous work [9]). Of course if it is so this directly implies the non-correctness of equation $E = m.c^2$.

In next part, we will use the correct mass definition obtained in referred reference [9], for deriving new and exact definitions of Energy and Momentum.

III. ENERGY DERIVATION FROM NEW DEFINITION OF MASS.

From previous work, the correct and unique definition of mass was obtained by working under the Local Lorentz Transformations (LLT) and by the application of the Angular Momentum Conservation Law to an inertial curvilinear movement of a mass attracted by another one [9]. Such mass definition was derived and established without any assumption. As it was referred before, this expression was:

$$m = \frac{M_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \qquad (8)$$

It is noteworthy that Einstein also obtained the same relationship, under a different procedure and called it "longitudinal mass" [1] to distinguish it from transverse mass definition. Later, the longitudinal definition of mass little by little was left out, until apparently it was discarded from his work..(!)

Relativistic Mass, as a unique definition, expressed in equation (8) came up as product of a simple and rigorous derivation, without assumptions, and therefore it should not be considered in any case as a proposal, assumption or suggested definition.

It can be observed that the new relativistic mass definition ratifies the dependence of the body's mass on its velocity, as it is reflected by equation (8). In the next section a different definition of Energy directly derived from the referred new mass definition will be obtained.

A. New Definition of Kinetic Energy

Let's consider a mass that moves describing a curvilinear path at velocity $\mathbf{v}$. As it was previously indicated, kinetic energy expression is given by:

$$\begin{aligned} dK &= \mathbf{F} \bullet d\mathbf{s} = \frac{d(m.\mathbf{v})}{dt} \bullet d\mathbf{s} = d(m.\mathbf{v}).\frac{d\mathbf{s}}{dt} = d(m.\mathbf{v}).\mathbf{v} \\ dK &= m.d\mathbf{v} \bullet \mathbf{v} + \mathbf{v} \bullet \mathbf{v}.dm = m.dv.v + v^2.dm \end{aligned} \tag{9}$$

Observe that in (8), M_0, the rest mass, is a constant magnitude and the only variable is its speed, v. Accordingly, the differential of mass can be derived as follows:

$$m = \frac{M_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} = \frac{M_0.c^3}{\left(c^2 - v^2\right)^{\frac{3}{2}}} \tag{10}$$

$$dm = 3.M_0.c^3.\frac{v.dv}{\left(c^2 - v^2\right)^{\frac{5}{2}}} = 3.\frac{M_0.c^3}{\left(c^2 - v^2\right)^{\frac{3}{2}}}.\frac{v.dv}{\left(c^2 - v^2\right)} = 3.m.\frac{v.dv}{\left(c^2 - v^2\right)}$$

$$dm = 3.m.\frac{v.dv}{\left(c^2 - v^2\right)} \qquad dm = 3.M_0.c^3.\frac{v.dv}{\left(c^2 - v^2\right)^{\frac{5}{2}}} \tag{11}$$

A similar procedure to that used in previous section is repeated here for obtaining the new expression of energy. Operating on the <u>first</u> equation in (11) and trying to recreate $dK = m.dv.v + v^2.dm$, the last energy equation in (9), it follows that:

$$\frac{c^2.dm}{3} - \frac{v^2.dm}{3} = m.v.dv \qquad \Rightarrow \qquad \frac{c^2.dm}{3} = m.v.dv + \frac{v^2.dm}{3}$$

By adding, $\frac{2.v^2.dm}{3}$, to both members of the last relationship, we can construct the known kinetic energy expression:

$$\frac{c^2.dm}{3} + \frac{2.v^2.dm}{3} = m.v.dv + \frac{v^2.dm}{3} + \frac{2.v^2.dm}{3} = m.v.dv + v^2.dm = dK$$

$$\Rightarrow \quad dK = \frac{c^2.dm}{3} + \frac{2.v^2.dm}{3} \tag{12}$$

Substituting the <u>second</u> expression of dm that appears in (11) in the second term of the right member of equation (12), it follows:

$$dK = \frac{c^2.dm}{3} + 2.v^2.\frac{M_0.c^3.v.dv}{\left(c^2 - v^2\right)^{\frac{5}{2}}} \tag{13}$$

In this way we have in equation (13) the first term at right depending only on the mass m, and the second one depending only on the velocity v.

Before continuing, let's now refer for a moment to the Earth movement around the Sun, for detecting the different presentations mass can have (this can be thought referred to any other general case of two attracting masses). Let the point for starting measurements be Perihelion (closest point between Earth and Sun). At that moment, Earth has a velocity $V_0 \neq 0$, and an "initial" mass denoted by m_0, which is different from the rest mass, M_0. In fact, $m_0 = \frac{M_0}{\left(1 - \frac{V_0^2}{c^2}\right)^{\frac{3}{2}}}$. When Earth occupies any other position on its elliptical path, it will have another generic velocity v, and another generic mass value, denoted as: $m = \frac{M_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}}$.

After this parenthesis, where the mass can be recognized (in general, in a curved movement) in three distinct manners, let's do the integration of the differential of kinetic energy indicated in equation (13), between (V_0, m_0) and (v, m). Thus, after some simplifications, the general (rectilinear and/or curvilinear) dynamic equation for a change in Kinetic Energy is given by:

$$K - K_0 = m.\left(2.v^2 - c^2\right) - m_0.\left(2.V_0^2 - c^2\right) \tag{14}$$

See that K_0 is that kinetic energy at V_0, when Earth is at perihelion.

By operating upon equation (14), in order to have a suitable expression for later expanding their elements in binomial series:

$$K - K_0 = m.v^2 + m.\left(v^2 - c^2\right) - m_0.V_0^2 - m_0.\left(V_0^2 - c^2\right)$$

$$K - K_0 = m.v^2 - m_0.V_0^2 - m.c^2\left(1 - \frac{v^2}{c^2}\right) + m_0.c^2\left(1 - \frac{V_0^2}{c^2}\right)$$

Reordering and Substituting by mass expressions:

$$K - K_0 = \frac{M_0.v^2}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} - \frac{M_0.V_0^2}{\left(1 - \frac{V_0^2}{c^2}\right)^{\frac{3}{2}}} - \frac{M_0.c^2}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}} + \frac{M_0.c^2}{\left(1 - \frac{V_0^2}{c^2}\right)^{\frac{1}{2}}}$$

Expanding in binomial series, as before, by using:

$$(1 + x)^{-\frac{1}{2}} = 1 - \frac{1}{2}.x + \frac{1.3}{2.4}.x^2 - \frac{1.3.5}{2.4.6}.x^3 + ... \quad -1 < x \leq 1$$

$$(1 + x)^{-\frac{3}{2}} = 1 - \frac{3}{2}.x + \frac{3.5}{2.4}.x^2 - \frac{3.5.7}{2.4.6}.x^3 + ... \quad -1 < x \leq 1$$

Putting $v << c$ for reducing to the Newtonian case where $m = \frac{M_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \cong M_0$, and discarding those terms divided by c^2, raised to any exponent, it is obtained:

$$K - K_0 = M_0.\left[\left(v^2 + ...\right) - \left(V_0^2 + ...\right) - \left(c^2 + \frac{1}{2}.v^2 + ...\right) + \left(c^2 + \frac{1}{2}.V_0^2 + ...\right)\right]$$

The change in kinetic energy reduces consistently to the known Newtonian expression:

$$K - K_0 \cong \frac{1}{2}.m.v^2 - \frac{1}{2}.m.V_0^2 \quad (15)$$

The Kinetic Energy of a body in any curvilinear movement K, when starting from rest: $V_0 = 0$, and, $m_0 = M_0$, until it gets a velocity v, and mass m, equation (14) reduces consistently to:

$$K = m.\left(2.v^2 - c^2\right) + M_0.c^2 \quad (16)$$

Operating on this equation, in order to check its consistency:

$$K = m.v^2 + m.\left(v^2 - c^2\right) + M_0.c^2 = m.v^2 - m.c^2.\left(1 - \frac{v^2}{c^2}\right) + M_0.c^2$$

$$K = \frac{M_0.v^2}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} - \frac{M_0.c^2}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}} + M_0.c^2 = M_0.\left[\left(v^2 + \ldots\right) - \left(c^2 + \frac{1}{2}v^2 + \ldots\right) + c^2\right]$$

This equation also reduces, for $v << c$, and $M_0 \cong m$, to the Newtonian expression of kinetic energy of a body starting from rest:

$$K \cong \frac{1}{2}.m.v^2 \tag{17}$$

But, nature is very tricky because as we have seen before Einstein's expression of total energy, $E - E_0 = m.c^2 - m_0.c^2$, also meets all these simplifications!. Those Einstein's blurred results led to the general acceptation of transverse mass definition and energy equation $E = m.c^2$. The crucial point in this confusion was finally solved by the correct and unique definition of mass in [9]. As it can be observed, the new definition (14) applies for any body in any situation, with or without rest mass, and takes into account, noticeably and explicitly, all the velocities involved.

B. New Definition of Total Energy

Let's continue with our task of construction all the new definitions of energy. Remembering that Total Energy, for a free particle, is the summation of Kinetic Energy starting from rest (in its simpler presentation), previously obtained, plus Internal Energy, it follows:

$$E = K + M_0.c^2 = m.\left(2.v^2 - c^2\right) + M_0.c^2 + M_0.c^2 = 2.M_0.c^2 - m.\left(c^2 - 2.v^2\right) \tag{18}$$

Let's check. By doing, $v = 0 \Rightarrow m = M_0$, then Total Energy is reduced consistently to the internal enegy, $E = M_0.c^2$. Thus, the equation (18) is a general expression for any body with moving mass m, starting from rest.

As it is well-known, photons do not have rest mass. Also, they do not exist for any other speed different from c; but when they exist, each one of them does have a mass m and a constant velocity equal to c. So, applying these conditions into equation (18) it is obtained that the energy of a Photon is $E = m..c^2$. ***Thus, from this development, is concluded that equation*** $E = m..c^2$***, is valid only for photons or particles with null rest mass.*** It is not valid for bodies with non-null rest masses. Then, the general Total Energy expression for any body, moving and starting from $V_0 = 0$, is that of the equation (18), and the particular version for photons is $E = m..c^2$.

A comparison between $E = 2.M_0.c^2 - \frac{M_0.c^2}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}}.\left(1-2.\frac{v^2}{c^2}\right)$ and $E = \frac{M_0.c^2}{\sqrt{1-\frac{v^2}{c^2}}}$, the new Energy definition and Einstein's Total Energy in function of transverse mass, considered as if it were valid for particles with non-null rest mass, respectively, is displayed below:

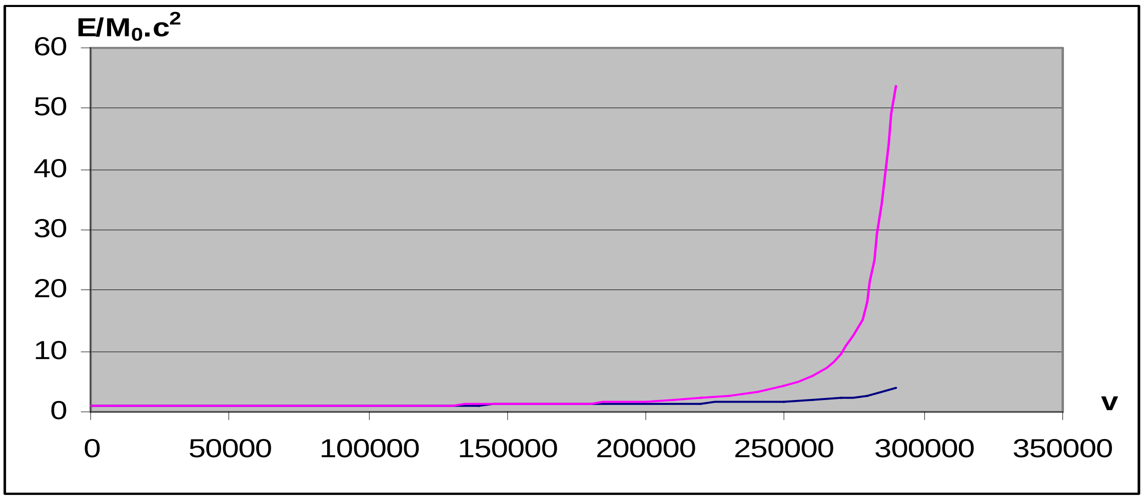

Fig. No. 1. Comparing the new definition of Energy with Einstein's Energy definition. Ordinates: $E / M_0.c^2$; Abcissa: Velocity v. Einstein's Energy $E=m.c^2$ In black. The new definition of Energy $E=2.M_0.c^2-m.\left(c^2-2.v^2\right)$ in red.

As it can be observed, both curves are very close until the body is moving at approximately a velocity of 200.000 KM/sec. From this graph, in our opinion it will be difficult to design an experiment to detect experimentally the coincidence of the correct values with Einstein's or ours. May be, if it were possible, the experiment should be conducted instead to detect the difference between the Einstein's mass definition, $m = \frac{M_0}{\sqrt{1-\frac{v^2}{c^2}}}$, and that obtained in [9], $m = \frac{M_0}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}}$, in where the difference is little bit more significant at smaller speeds than the previous ones. As an illustration a comparative graph is given next:

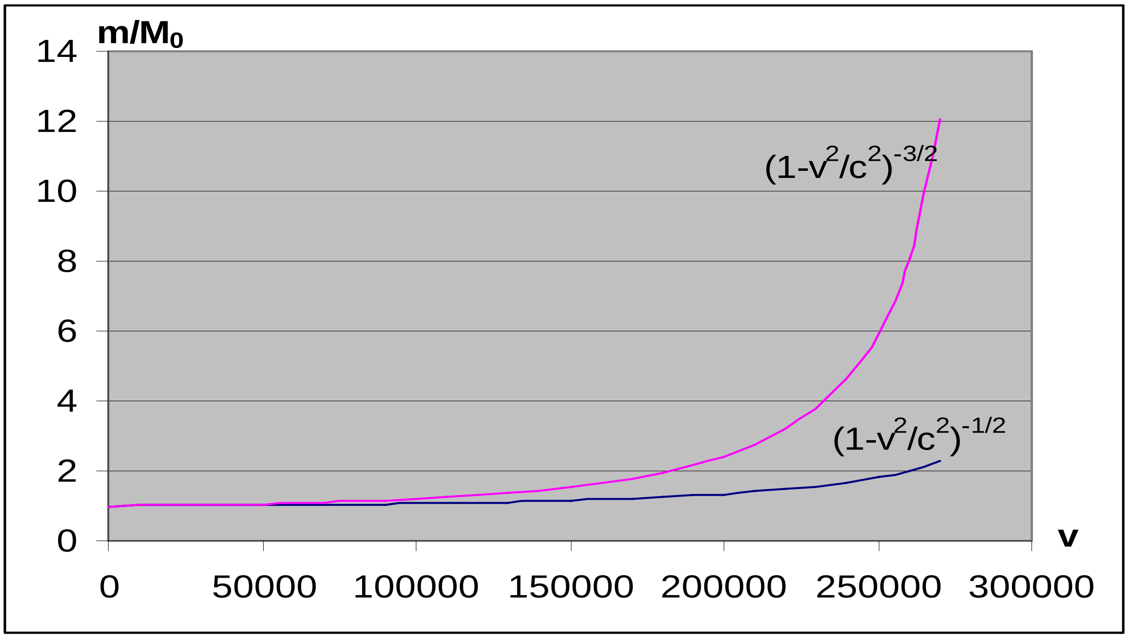

Fig. No. 2. Comparing new definition of mass with Einstein's mass definition (Black: Einstein's; Red: Ours)

C. New Definition of Linear Momentum

Likewise as Energy was redefined, we will do the same with Linear Momentum. By using the new mass definition, Momentum will be then:

$$p = m.v = \frac{M_0.v}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \tag{19}$$

By substituting rest mass, $M_0 = m.\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}$, in equation (18), Energy and mass are related as follows:

$$E = 2.M_0.c^2 - m..\left(c^2 - 2.v^2\right) = 2.m.\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}} - m..\left(c^2 - 2.v^2\right)$$

Introducing mass as function of Energy in (18), we have another expression for linear momentum :

$$p = m.v = \frac{E}{2.c^2.\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}} - \left(c^2 - 2.v^2\right)}.v \qquad (20)$$

For instance, Momentum for the case of Photon, where, $v = c$ and $M_0 = 0$, is reduced to the known relationship: $p = \frac{E}{c}$ (21)

An expression of the Energy-Momentum can also be encountered, and it is:

$$E = 2.M_0.c^2 + p.v - c.\sqrt{M_0^2.c^2 + p^2.\left(1-\frac{v^2}{c^2}\right)^2} \qquad (22)$$

Let's check. By substituting $p.v = m.v^2$ and the Momentum expression (19) inside the square root in (22) and simplifying, the original expression of energy (18) is obtained:

$$E = 2.M_0.c^2 + m..v^2 - m.c^2.\left(1-\frac{v^2}{c^2}\right) = 2.M_0.c^2 + m.v^2 - m.c^2 + m.v^2 = 2.M_0.c^2 - m..\left(c^2 - 2.v^2\right)$$

As it has been shown, the new definitions for Mass, Energy and Momentum are consistent with known expressions as for "modern" (for photons) as for Newtonian physics.

IV. CONCLUSION

Although the quantitative gain in precise measurements of the energy that can be obtained with The new Energy definitions are not so significant, because Einstein's energy expressions are a good approach in the most of practical cases to experimental values, in our opinion the theoretical corrections informed in original Franco's work for the definitions of mass and Energy given by Einstein are very relevant [10], from a conceptual point of view in theoretical physics. May be, further measurements of matter characteristics, others than Energy, could lead to find relevant differences favoring the new dynamical definitions reliant on new mass and energy definitions given in this and in the previous work.

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