

# Time as a Vector and Vectorial Lorentz Transformations

J G Quintero <sup>D1</sup> and J A Franco <sup>R2</sup>

**ABSTRACT:** It has been shown in previous review that inside Lorentz Transformations (LT) there were some statements coming from the “common sense” or from “isotropy postulate” or from the “geometry of the problem”,  $y' = y$  and  $z' = z$ , which we will demonstrate that they are really assumptions and worst of all, needless and therefore groundless. Because of such assumptions, Lorentz Transformations (LT) depend on the body’s spatial orientation, i.e. the well-known transverse and longitudinal transformations of magnitudes, characterized by different scaling factors. In this review we show, following Franco’s procedure, that development of LT without assumptions brings about new Lorentz Transformations that do not depend on spatial orientation and reveals a vectorial behavior of time.

**KEYWORDS:** Special Relativity, Relativistic Mass, Relativistic Energy and Relativistic Momentum.

## I. TIME BEHAVES AS A SPATIAL VECTOR IN LORENTZ TRANSFORMATIONS

We are going to check that this is true. In this section we will depict the way how we were forced to arrive at the following concept of time: “Time is not only different for observers with distinct inertial movements, but additionally, it behaves between them as a vector, with **spatial components**”. As it will be observed, this concept will not result from any assumption or from any hypothesis. Instead, the vector structure will be deduced from the analysis of time’s obtained expressions, for one, two, three (or more, if it were necessary) **spatial** dimensions. The two-dimensional general case presented in the next **Fig. 1**, is a very illustrative example to clarify this statement:

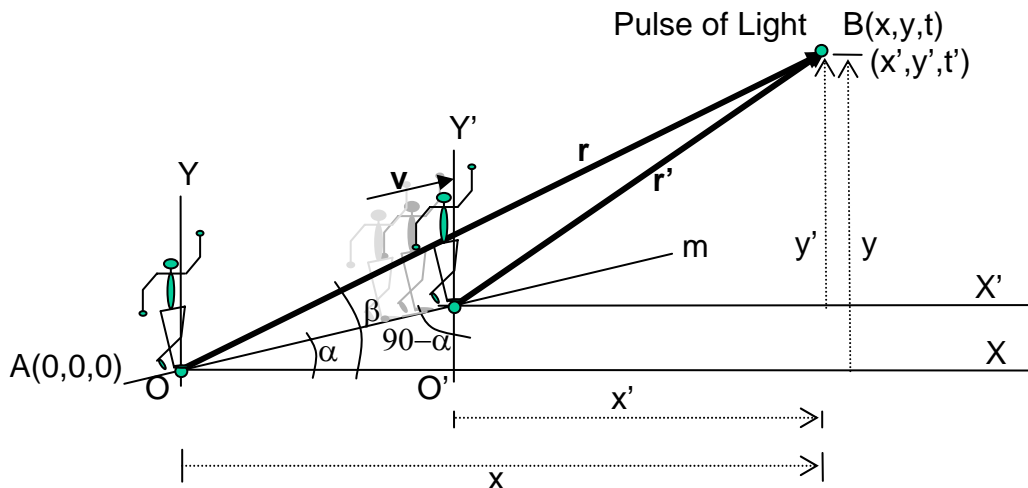


Fig. 1

<sup>1</sup>Independent Researcher, Caracas, Venezuela, [Journal.of.VR@hotmail.com](mailto:Journal.of.VR@hotmail.com)

<sup>2</sup>Independent Researcher, Caracas, Venezuela, [jafrancor@yahoo.com](mailto:jafrancor@yahoo.com)

Referring to **Fig. 1** when  $O'$ , moving along an inclined line  $m$ , and  $O$  coincide, a light pulse is sent in any direction. By defining  $\alpha$ , as the angle between line  $m$ , and  $X$  axis, the following equations hold:

$$\begin{aligned} x^2 + y^2 &= c^2 \cdot t^2 & \text{for,} & & x' &= k \cdot (x - v \cdot t \cdot \cos \alpha) \\ x'^2 + y'^2 &= c^2 \cdot t'^2 & & & y' &= k \cdot (y - v \cdot t \cdot \sin \alpha) \end{aligned} \quad (1)$$

Based on these relationships, by substituting, working on and grouping properly, we obtain:

$$\begin{aligned} c^2 \cdot t'^2 &= x'^2 + y'^2 = k^2 \cdot [(x - v \cdot t \cdot \cos \alpha)^2 + (y - v \cdot t \cdot \sin \alpha)^2] \\ c^2 \cdot t'^2 &= k^2 \cdot [(x^2 + y^2) + [v^2 \cdot (t \cdot \cos \alpha)^2 + v^2 \cdot (t \cdot \sin \alpha)^2] - 2 \cdot v \cdot x \cdot (t \cdot \cos \alpha) - 2 \cdot v \cdot y \cdot (t \cdot \sin \alpha)] \\ c^2 \cdot t'^2 &= k^2 \cdot \{c^2 \cdot t^2 + v^2 \cdot (t)^2 - 2 \cdot v \cdot [x \cdot (t \cdot \cos \alpha) + y \cdot (t \cdot \sin \alpha)]\} \end{aligned}$$

Substituting:  $c^2 \cdot t^2 \equiv c^2 \cdot t^2 \cdot (\sin^2 \alpha + \cos^2 \alpha)$ , and  $v^2 \cdot t^2 \equiv v^2 \cdot \frac{x^2 + y^2}{c^2}$ ; and grouping, we get:

$$\begin{aligned} c^2 \cdot t'^2 &= k^2 \cdot \{[c^2 \cdot (t \cdot \cos \alpha)^2 - 2 \cdot v \cdot x \cdot (t \cdot \cos \alpha) + v^2 \cdot \frac{x^2}{c^2}] + [c^2 \cdot (t \cdot \sin \alpha)^2 - 2 \cdot v \cdot y \cdot (t \cdot \sin \alpha) + v^2 \cdot \frac{y^2}{c^2}]\} \\ c^2 \cdot t'^2 &= k^2 \cdot [(c \cdot t \cdot \cos \alpha - \frac{v}{c} \cdot x)^2 + (c \cdot t \cdot \sin \alpha - \frac{v}{c} \cdot y)^2] = c^2 \cdot k^2 \cdot [(t \cdot \cos \alpha - \frac{v}{c^2} \cdot x)^2 + (t \cdot \sin \alpha - \frac{v}{c^2} \cdot y)^2] \end{aligned}$$

From the last relationship, it is obtained the following expression for time:

$$t'^2 = k^2 \cdot [(t \cdot \cos \alpha - \frac{v}{c^2} \cdot x)^2 + (t \cdot \sin \alpha - \frac{v}{c^2} \cdot y)^2]$$

By observing carefully the right hand side of the previous expression, it reminds us the module of a vector. Thus, as it is suggested, the previous modular expression can be re-organized into its corresponding two-dimensional vectorial structure, in the following way:

$$\begin{aligned} \mathbf{t}' &= k \cdot \left[ \left( t \cdot \cos \alpha - \frac{v}{c^2} \cdot x \right) \mathbf{i} + \left( t \cdot \sin \alpha - \frac{v}{c^2} \cdot y \right) \mathbf{j} \right] = k \cdot \left[ t \cdot \cos \alpha \cdot \mathbf{i} - \frac{v}{c^2} \cdot x \cdot \mathbf{i} + t \cdot \sin \alpha \cdot \mathbf{j} - \frac{v}{c^2} \cdot y \cdot \mathbf{j} \right] \\ \mathbf{t}' &= k \cdot \left[ (t \cdot \cos \alpha \cdot \mathbf{i} + t \cdot \sin \alpha \cdot \mathbf{j}) - \frac{v}{c^2} \cdot (x \cdot \mathbf{i} + y \cdot \mathbf{j}) \right] = k \cdot \left[ (t_x \cdot \mathbf{i} + t_y \cdot \mathbf{j}) - \frac{v}{c^2} \cdot (x \cdot \mathbf{i} + y \cdot \mathbf{j}) \right] \end{aligned}$$

$$\text{Thus, by defining: } \left\{ \begin{array}{l} t_x = t \cdot \cos \alpha \\ t_y = t \cdot \sin \alpha \\ \mathbf{t} = t_x \cdot \mathbf{i} + t_y \cdot \mathbf{j} \end{array} \right\} \text{ and } \left\{ \begin{array}{l} t'_x = k \cdot \left( t_x - \frac{v}{c^2} \cdot x \right) \\ t'_y = k \cdot \left( t_y - \frac{v}{c^2} \cdot y \right) \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \mathbf{t}' = k \cdot \left( \mathbf{t} - \frac{v}{c^2} \cdot \mathbf{r} \right) \\ \mathbf{r}' = k \cdot (\mathbf{r} - v \cdot \mathbf{t}) \end{array} \right\} \quad (2)$$

It can be realized that this vector structure of time can be easily obtained for any number of dimensions by repeating this same procedure. For instance:

In the three-dimensional case, see the next **Fig. 2**, the following relationships hold:

$$\begin{aligned}
 x^2 + y^2 + z^2 &= c^2 \cdot t^2 & x' &= k \cdot (x - v \cdot t \cdot \cos \alpha \cdot \cos \beta) & t_x &= t \cdot \cos \alpha \cdot \cos \beta \\
 x'^2 + y'^2 + z'^2 &= c^2 \cdot t'^2 & y' &= k \cdot (y - v \cdot t \cdot \sin \alpha \cdot \cos \beta) & \text{By defining: } t_y &= t \cdot \sin \alpha \cdot \cos \beta \\
 & & z' &= k \cdot (z - v \cdot t \cdot \sin \beta) & t_z &= t \sin \beta
 \end{aligned}$$

Following a similar procedure to that previously used is obtained again the familiar vector structure expression of time for three (or for any number of) dimensions:

$$\boxed{t'^2 = k^2 \cdot \left[ \left( t_x - \frac{v}{c^2} \cdot x \right)^2 + \left( t_y - \frac{v}{c^2} \cdot y \right)^2 + \left( t_z - \frac{v}{c^2} \cdot z \right)^2 \right]} \Rightarrow \begin{aligned} \mathbf{t}' &= k \cdot \left( \mathbf{t} - \frac{v}{c^2} \cdot \mathbf{r} \right) \\ \mathbf{r}' &= k \cdot (\mathbf{r} - v \cdot \mathbf{t}) \end{aligned} \quad (2)$$

All these results lead consistently to consider the behavior of time as a vector when it is referred to observers located in systems with different inertial movements.

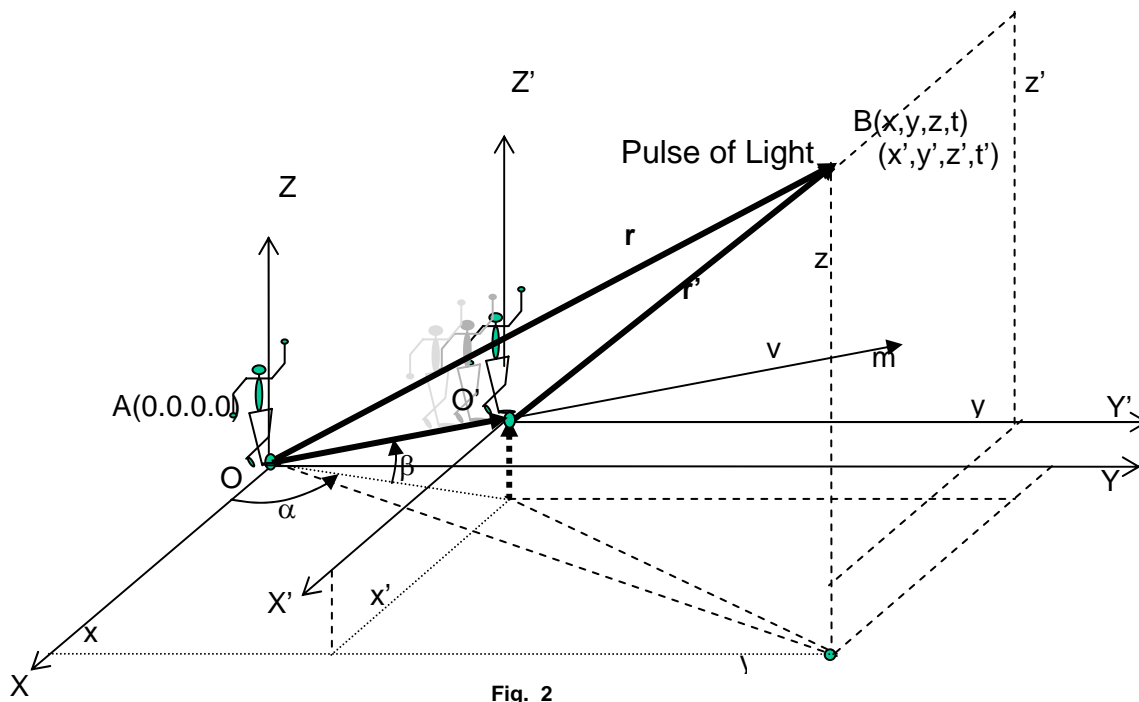


Fig. 2

Certainly, it can be shown that the invariance of the space-time interval of the Special Theory of Relativity is preserved for **any number of dimensions**, which means that this new presentation of LT are valid not only for light but for any projectile, i.e.: For  $k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$  and substituting general

coordinate components in the expression of the space-time interval,  $c^2 \cdot t'^2 - r'^2$ :

$$\begin{aligned}
c^2.t'^2 - r'^2 &= \sum_{j=1}^N c^2.k^2 \left( t_j - \frac{v}{c}.x_j \right)^2 - k^2 \cdot \sum_{j=1}^N (x_j - v.t_j)^2 = k^2 \cdot \sum_{j=1}^N \left[ \left( c.t_j - \frac{v}{c}.x_j \right)^2 - (x_j - v.t_j)^2 \right] = \\
&= k^2 \cdot \sum_{j=1}^N \left[ \left( c^2.t_j^2 - 2.v.t_j.x_j + \frac{v^2}{c^2}.x_j^2 \right) - (x_j^2 - 2.v.t_j.x_j + v^2.t_j^2) \right] = k^2 \cdot \sum_{j=1}^N \left[ c^2.t_j^2 + \frac{v^2}{c^2}.x_j^2 - x_j^2 - v^2.t_j^2 \right] \\
&= k^2 \cdot \sum_{j=1}^N \left[ c^2.t_j^2 + \frac{v^2}{c^2}.x_j^2 - x_j^2 - v^2.t_j^2 \right] = k^2 \cdot \sum_{j=1}^N \left[ c^2.t_j^2 - v^2.t_j^2 + \frac{v^2}{c^2}.x_j^2 - x_j^2 \right] = \\
&= k^2 \cdot \left( 1 - \frac{v^2}{c^2} \right) \cdot \sum_{j=1}^N (c^2.t_j^2 - x_j^2) = \sum_{j=1}^N (c^2.t_j^2 - x_j^2) = \sum_{j=1}^N (c^2.t_j^2) - \sum_{j=1}^N (x_j^2) \Rightarrow c^2.t'^2 - r'^2 = c^2.t^2 - r^2
\end{aligned}$$

So, this procedure demonstrates that vector character of time is not the result of any hypothesis; it comes directly from observing vector properties clearly present inside transformations relating measurements of both inertial observers. It can also be observed, from an epistemological point of view, that time as a spatial vector forms its direction by taking it from the vector velocity  $v$  of the moving system  $O'$ , leaving such parameter with a scalar character and functioning as part of a scaling factor. This behavior of the velocity can be understood due to both observers are on the same inclined line, which will imply this scalar character of  $v$ . Another epistemological characteristic of vector time is its dependence on spatial coordinates  $x$ ,  $y$  and  $z$ , which means that **it is not an additional independent vector** to our known three-dimensional-spatial universe, a characteristic that appears remarkable because it differs from the Minkowski's four dimensions universe (time as a fourth independent dimension) introduced by Einstein. This would mean that for obtaining exact results we are allowed to continue working within our familiar three spatial dimensions in this study, and that magnitudes can continue being defined as in classical physics, but from a modern and relativistic point of view. By observing the previous obtained **results** we arrive at the following concept of time: Time is forced to behave as a vector with spatial components in each coordinate, when it appears inside an analysis without assumptions which we are going to name now on, Vectorial Lorentz Transformations (VLT), but it can appear behaving as a scalar value when it is not an element of a transformation such as VLT in the way we always have known it: as a sequential meter of events. But moreover, time can also be considered as a vector in the natural way it was referred to in [4]. In accordance with this idea and it perfectly applies to our work, Hongbao Ma says: "*this three dimensional time concept is obtained from the mathematical conception rather than the ontological existence. Mathematical results are at the epistemological level*" [4]. It is worth to mention that a similar and rigorous presentation of time as a vector, very close to the way we have presented here can be seen in the work done by Bernard Guy [5], published in 2001.

## II. VECTORIAL LORENTZ TRANSFORMATIONS

By considering time as having the properties of a vector with components on the spatial coordinates, when reflected within the relation between inertial observers with different movements, let's formally obtain the vectorial version for the Lorentz transformations (VLT). So, now we will refer in general to the three-dimensional case, or for further research it could be thought in an n-dimensional case, (see **Section I**), where system  $O'$  moves on a general inclined line and both observers measure the light

pulse radio-vectors  $\mathbf{r}$ ,  $\mathbf{r}'$  and vectorial times  $\mathbf{t}$ ,  $\mathbf{t}'$ . Vectors from now on will be written as boldface letters. The relationships in VLT, previously seen, are easily obtained from **Fig. 3**,

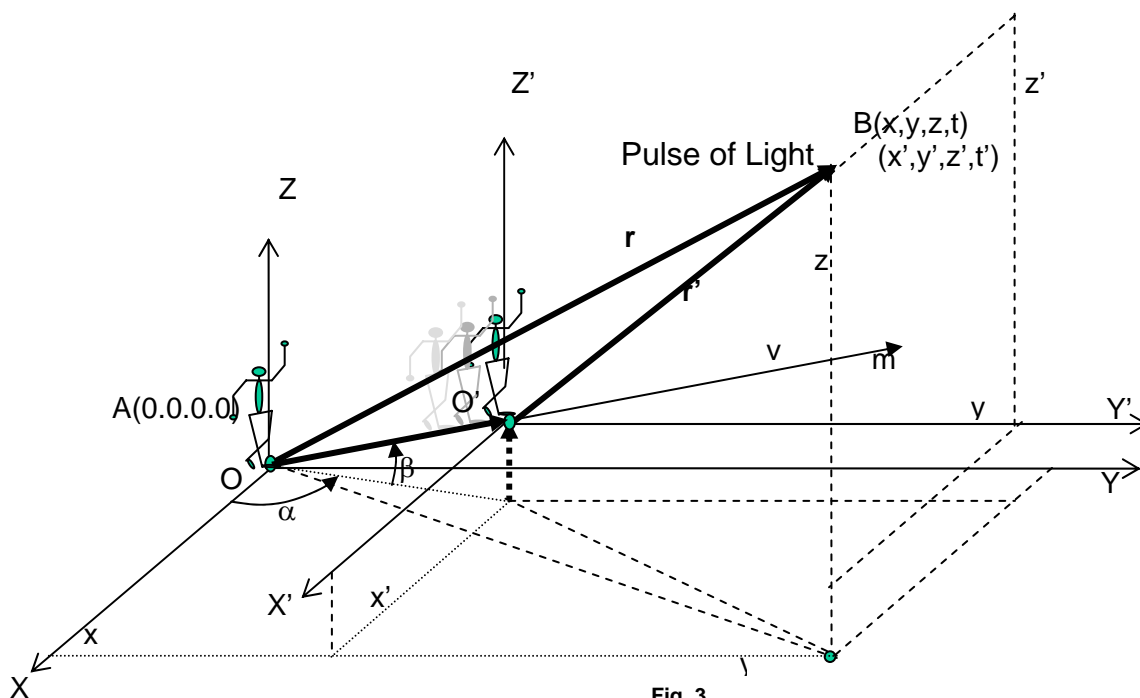


Fig. 3

$$\mathbf{r} = c \cdot \mathbf{t} \quad \mathbf{r}' = c \cdot \mathbf{t}' \quad \mathbf{t}' = \frac{\mathbf{r}'}{c} \quad \mathbf{t} = \frac{\mathbf{r}}{c}$$

$$\left\{ \begin{array}{l} \mathbf{r}' = k(\mathbf{r} - v \cdot \mathbf{t}) \Rightarrow c \cdot \mathbf{t}' = k \cdot \mathbf{t} \cdot (c - v) \\ \mathbf{r} = k(\mathbf{r}' + v \cdot \mathbf{t}') \Rightarrow c \cdot \mathbf{t} = k \cdot \mathbf{t}' \cdot (c + v) \end{array} \right\} \Rightarrow c^2 \cdot \mathbf{t} \cdot \mathbf{t}' = k^2 \cdot \mathbf{t}' \cdot \mathbf{t} \cdot (c^2 - v^2) \Rightarrow k^2 = \frac{1}{1 - \frac{v^2}{c^2}}$$

$$\mathbf{r}' = \frac{\mathbf{r} - v \cdot \mathbf{t}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow c \cdot \mathbf{t}' = \frac{c \cdot \mathbf{t} - v \cdot \frac{\mathbf{r}}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \mathbf{t}' = \frac{\mathbf{t} - \frac{v}{c^2} \cdot \mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \mathbf{u}' = \frac{d\mathbf{r}'}{dt'} = \frac{d\mathbf{r} - v \cdot dt}{\left| dt - \frac{v}{c^2} \cdot d\mathbf{r} \right|} \quad (3)$$

We have previously seen that the following equality also holds as invariant for VLT:  $c^2 \cdot t'^2 - r'^2 = c^2 \cdot t^2 - r^2$ . This means that the cinematic VLT, composed by expressions,  $\mathbf{r}'$ ,  $\mathbf{t}'$  and  $\mathbf{u}'$  in (3), are generally valid for a light pulse or for any projectile moving at any speed less than  $c$ . As a check, the Jacobian matrix for any value of variables, becomes symmetric and equal to one, i.e., Letting  $x^i$ , be the variables measured by O, and  $\bar{x}^j$  with bar,  $\bar{x}^j$ , be those measured by O', for

$i, j = 1, 2, 3$ , and for  $k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ , we have:

$$\left\{ \begin{array}{l} \mathbf{r}' = k.(\mathbf{r} - v.\mathbf{t}) \quad \mathbf{r} = k.(\mathbf{r}' + v.\mathbf{t}') \\ \mathbf{t}' = k.\left(\mathbf{t} - \frac{v}{c^2}.\mathbf{r}\right) \quad \mathbf{t} = k.\left(\mathbf{t}' + \frac{v}{c^2}.\mathbf{r}'\right) \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \left(\frac{\partial x^i}{\partial \bar{x}^j}\right) = \begin{vmatrix} k & -k.v \\ -k.\frac{v}{c^2} & k \end{vmatrix} \\ \left(\frac{\partial \bar{x}^j}{\partial x^i}\right) = \begin{vmatrix} k & +k.v \\ +k.\frac{v}{c^2} & k \end{vmatrix} \end{array} \right\} \left(\frac{\partial x^i}{\partial \bar{x}^j}\right) = \left(\frac{\partial \bar{x}^j}{\partial x^i}\right) = 1$$

This is valid for any set of components:  $\left\{ \begin{array}{l} y' = k.(y - v.t_y) \quad y = k.(y' + v.t'_y) \\ t'_y = k.\left(t_y - \frac{v}{c^2}.y\right) \quad t_y = k.\left(t'_y + \frac{v}{c^2}.y'\right) \end{array} \right\}$

A final remark on the procedure previously presented: It was not done any assumption for obtaining the VLT presented in (2). Thus, because these are vectorial relationships, they are generally valid for any number of dimensions. It is also opportune to say that the consistency of VLT with Maxwell Equations is also demonstrated in Annex, at the end of this section.

Let's obtain the general expressions for VLT in three dimensions using spherical coordinates (**Fig. 3**) and relationships appearing in (2). Allow  $\beta$  to be the angle between the inclined trajectory of  $O'$  and the plane  $XY$ ; and allow  $\alpha$  to be the angle formed by the projection of the inclined trajectory of  $O'$  on the plane  $XY$ , with the  $X$ -axis. When moving origin  $O'$  and fixed one  $O$  coincide, the light pulse is sent towards the space with generic components  $x, y, z$ , The general VLT of the vector time and that of the radio-vector of the pulse of light (or projectile), in three dimensions, become:

$$\begin{array}{l} x' = \frac{x - v.t.\cos\alpha.\cos\beta}{\sqrt{1 - \frac{v^2}{c^2}}} \\ y' = \frac{y - v.t.\sin\alpha.\cos\beta}{\sqrt{1 - \frac{v^2}{c^2}}} \\ z' = \frac{z - v.t.\sin\beta}{\sqrt{1 - \frac{v^2}{c^2}}} \end{array} \quad \begin{array}{l} t_x = t.\cos\beta.\cos\alpha \\ t_y = t.\cos\beta.\sin\alpha \\ t_z = t.\sin\beta \end{array} \quad \begin{array}{l} x' = \frac{x - v.t_x}{\sqrt{1 - \frac{v^2}{c^2}}} \\ y' = \frac{y - v.t_y}{\sqrt{1 - \frac{v^2}{c^2}}} \\ z' = \frac{z - v.t_z}{\sqrt{1 - \frac{v^2}{c^2}}} \end{array} \quad (4)$$

$$t' = \frac{\left| \mathbf{t} - \frac{v}{c^2}.\mathbf{r} \right|}{\sqrt{1 - \frac{v^2}{c^2}}} = \sqrt{\frac{(t_x - \frac{v}{c^2}.x)^2 + (t_y - \frac{v}{c^2}.y)^2 + (t_z - \frac{v}{c^2}.z)^2}{1 - \frac{v^2}{c^2}}} \quad (5)$$

The general expressions for the velocities of the pulse of light or any projectile are obtained from the previous ones:

$$\begin{aligned}
 u'_x &= \frac{u_x - v \cdot \cos \alpha \cdot \cos \beta}{\sqrt{\left(\cos \alpha \cdot \cos \beta - \frac{v \cdot u_x}{c^2}\right)^2 + \left(\sin \alpha \cdot \cos \beta - \frac{v \cdot u_y}{c^2}\right)^2 + \left(\sin \beta - \frac{v \cdot u_z}{c^2}\right)^2}} \\
 u'_y &= \frac{u_y - v \cdot \sin \alpha \cdot \cos \beta}{\sqrt{\left(\cos \alpha \cdot \cos \beta - \frac{v \cdot u_x}{c^2}\right)^2 + \left(\sin \alpha \cdot \cos \beta - \frac{v \cdot u_y}{c^2}\right)^2 + \left(\sin \beta - \frac{v \cdot u_z}{c^2}\right)^2}} \\
 u'_z &= \frac{u_z - v \cdot \sin \beta}{\sqrt{\left(\cos \alpha \cdot \cos \beta - \frac{v \cdot u_x}{c^2}\right)^2 + \left(\sin \alpha \cdot \cos \beta - \frac{v \cdot u_y}{c^2}\right)^2 + \left(\sin \beta - \frac{v \cdot u_z}{c^2}\right)^2}}
 \end{aligned} \tag{6}$$

Now, let's particularize these general results to the conditions from where the original LT were obtained (**Fig. 4**). If we re-establish such conditions (the system O' moving along the X axis, and the light pulse sent to space), i.e., for  $\alpha=\beta=0$ , we will obtain the VLT version of the original Lorentz transformations:

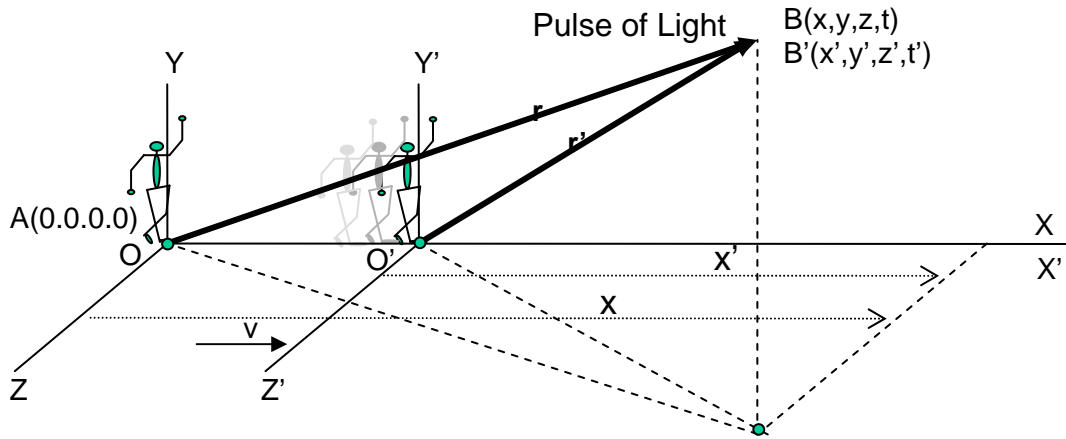


Fig. 4

$$\boxed{
 \begin{aligned}
 x' &= \frac{x - v \cdot t}{\sqrt{1 - \frac{v^2}{c^2}}}; & y' &= \frac{y}{\sqrt{1 - \frac{v^2}{c^2}}}; & z' &= \frac{z}{\sqrt{1 - \frac{v^2}{c^2}}}
 \end{aligned}
 }
 \quad
 \begin{aligned}
 t_x &= t \\
 t_y &= 0 \\
 t_z &= 0
 \end{aligned}
 \quad
 t' = \sqrt{\frac{\left(t_x - \frac{v}{c^2} \cdot x\right)^2 + \left(\frac{v}{c^2} \cdot y\right)^2 + \left(\frac{v}{c^2} \cdot z\right)^2}{1 - \frac{v^2}{c^2}}} \tag{7}$$

$$\begin{array}{|l}
 \boxed{u'_x = \frac{u_x - v}{\sqrt{\left(1 - \frac{v.u_x}{c^2}\right)^2 + \left(\frac{v.u_y}{c^2}\right)^2 + \left(\frac{v.u_z}{c^2}\right)^2}} \\
 \boxed{u'_z = \frac{u_z}{\sqrt{\left(1 - \frac{v.u_x}{c^2}\right)^2 + \left(\frac{v.u_y}{c^2}\right)^2 + \left(\frac{v.u_z}{c^2}\right)^2}}
 \end{array}
 \quad
 \begin{array}{|l}
 \boxed{u'_y = \frac{u_y}{\sqrt{\left(1 - \frac{v.u_x}{c^2}\right)^2 + \left(\frac{v.u_y}{c^2}\right)^2 + \left(\frac{v.u_z}{c^2}\right)^2}} \\
 u_x'^2 + u_y'^2 + u_z'^2 = u_x^2 + u_y^2 + u_z^2 = c^2
 \end{array}
 \tag{8}$$

Let's check the last relationship in (8), which is valid only for photons. In such equation is then implied that the velocity of light measured by any of the two observers should be the same,  $c$  (In general for any other projectile,  $u'^2 \neq u^2$ ), i. e., on the basis of which  $O$  measures,  $u_x^2 + u_y^2 + u_z^2 = c^2$ , then  $O'$  will measure:

$$\begin{aligned}
 u_x'^2 + u_y'^2 + u_z'^2 &= \frac{(u_x - v)^2 + u_y^2 + u_z^2}{\left(1 - \frac{v.u_x}{c^2}\right)^2 + \left(\frac{v.u_y}{c^2}\right)^2 + \left(\frac{v.u_z}{c^2}\right)^2} = \frac{u_x^2 - 2.v.u_x + v^2 + u_y^2 + u_z^2}{1 - \frac{2.v.u_x}{c^2} + \frac{v^2.u_x^2}{c^4} + \frac{v^2.u_y^2}{c^4} + \frac{v^2.u_z^2}{c^4}} = \\
 &= \frac{(u_x^2 + u_y^2 + u_z^2) - 2.v.u_x + v^2}{1 - \frac{2.v.u_x}{c^2} + \frac{v^2}{c^2} \cdot (u_x^2 + u_y^2 + u_z^2)} = \frac{c^2 - 2.v.u_x + v^2}{1 - \frac{2.v.u_x}{c^2} + \frac{v^2}{c^2}} = \frac{c^2 \cdot \left(1 - \frac{2.v.u_x}{c^2} + \frac{v^2}{c^2}\right)}{1 - \frac{2.v.u_x}{c^2} + \frac{v^2}{c^2}} = c^2
 \end{aligned}$$

And by using the particularized equations (1.4.5), the demonstration of the space-time interval invariance readily follows:

$$\begin{aligned}
 r'^2 - c^2.t'^2 &= \frac{(x - v.t)^2 + y^2 + z^2}{1 - \frac{v^2}{c^2}} - c^2 \cdot \frac{\left(t - \frac{v}{c^2}.x\right)^2 + \left(\frac{v}{c^2}.y\right)^2 + \left(\frac{v}{c^2}.z\right)^2}{1 - \frac{v^2}{c^2}} = \\
 r'^2 - c^2.t'^2 &= \frac{x^2 + y^2 + z^2 - 2.v.t.x + v^2.t^2 - c^2.t^2 + 2.v.t.x - \frac{v^2}{c^2} \cdot (x^2 + y^2 + z^2)}{1 - \frac{v^2}{c^2}} = \\
 r'^2 - c^2.t'^2 &= \frac{(x^2 + y^2 + z^2) \left[1 - \frac{v^2}{c^2}\right] - \left[1 - \frac{v^2}{c^2}\right] c^2.t^2}{1 - \frac{v^2}{c^2}} = r^2 - c^2.t^2
 \end{aligned}$$

When comparing equations (7) and (8), with the original LT equations, repeated in (9) and (10):

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \boxed{y' = y \quad z' = z} \quad t' = \frac{t - \frac{v \cdot x}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} = t \cdot \frac{1 - \frac{v}{c^2} \cdot u_x}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (9)$$

$$u'_x = \frac{u_x - v}{1 - \frac{v \cdot u_x}{c^2}} \quad \boxed{u'_y = \frac{u_y \cdot \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v \cdot u_x}{c^2}} \quad u'_z = \frac{u_z \cdot \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v \cdot u_x}{c^2}}} \quad (10)$$

The first thing we realize in equations (7) is that components  $y, z$ , measured by the fixed observer are different to those of  $y', z'$ , measured by the moving observer, thus, contradicting original LT's "statements" (assumptions). Additionally, we can observe that the expression of time in (8) is completely different to that of Lorentz in (9). And of course, the obtained expressions, according to this work, for velocity components  $u'_x, u'_y, u'_z$  in equations (8), are also different of those presented for LT in (10). By the way, recently J. H. Field following the canonical form of presentation of the LT in a detailed manner, shows as mathematically correct the assumptions  $y' = y$  and  $z' = z$  [6]. On the contrary, according to the current work these assumptions were shown to be groundless. Thus, in author's opinion, there are only two relevant possibilities for obtaining such disagreement: either isotropy postulate is not applicable for this configuration or postulate can't be applied in relativity. May be further research will answer this question.

Given that our procedure to obtain the vectorial transformations did not use any type of assumptions, it sufficiently demonstrates that in LT they were needless and therefore, LT canonical procedure is reduced to be only valid for one spatial dimension. Thus, its validity cannot be extrapolated to a general configuration.

It is conceivable that when in 1905 Einstein established his remarkable concept of the variation of mass with its velocity [2], he was actually looking for the one-to-one variation of physical magnitudes between classic and relativistic physics through Lorentz factors. At that time he already "had" the relationships for length, time, velocity and mass. So, Einstein probably got to consider Lorentz transformations (*not general, as we have shown previously*) as a central part of the SRT [3]. In an author's speculative opinion, Einstein later abandons SRT due to some observed inconsistencies and limitations of LT, and may be this was one of reasons he had for developing the General Theory of Relativity (GRT) trying to avoid such type of limitations in his research.

**REFERENCES**

[1] H. A. Lorentz. [\*"Electromagnetic phenomena in a system moving with any velocity less than that of light"\*](#) Proc. Acad. Sci. of Amsterdam, **6**, 1904.

[2] Albert Einstein. *Zur Elektrodynamik bewegter Körper*, Annalen der Physik 17, 1905, pp. 891-921. English version. *On the Electrodynamics of Moving Bodies*. <http://www.fourmilab.ch/etexts/einstein/specrel/www/>

- [3] Albert Einstein. *The Meaning of Relativity*, Fifth Edition, MJF Books, New York, 1956. Page 34.
- [4] Hongbao Ma. [The Nature of Time and Space](#) Nature and Science 1 (1) November 2003. Page 8, section 18.
- [5] Guy B. *The Duality of Space and Time and the Theory of Relativity*. HADRONIC JOURNAL SUPPLEMENT 16, 369-412 (2001). Hadronic Press Inc., Palm Harbor FL 34682, USA.
- [6] J. H. Field. *A New Kinematical Derivation of the Lorentz Transformation and the Particle Description of Light*. ArXiv: physics/0410262 v1 27 Oct 2004. <http://www.lanl.gov/abs/physics/0501043>.
- [7] J A Franco R, [Vectorial Lorentz Transformations](#). 2006. EJTP 9 (2006) 35-64..

## ANNEX 1

### CONSISTENCY OF VECTORIAL LORENTZ TRANSFORMATIONS AND INCONSISTENCY OF LORENTZ TRANSFORMATIONS

Given that we are working with vectors it is suitable to obtain a Wave Equation presentation in function of the light pulse radio-vector and time vector.

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \quad \Rightarrow \quad r = \sqrt{x^2 + y^2 + z^2} \quad \Rightarrow \quad \frac{\partial r}{\partial x} = \frac{x}{r}; \quad \frac{\partial r}{\partial y} = \frac{y}{r}; \quad \frac{\partial r}{\partial z} = \frac{z}{r};$$

In this way, each one of the components of the operator  $\nabla$  can be represented as depending on both vectors. Let's work to arrive at equations depending only on  $r$  and  $t$ . For this, we will develop the expressions of the components  $x$ ,  $y$ ,  $z$ :

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial r} \frac{\partial r}{\partial x} + \frac{\partial}{\partial t} \frac{\partial t}{\partial x}; \quad \frac{\partial}{\partial y} = \frac{\partial}{\partial r} \frac{\partial r}{\partial y} + \frac{\partial}{\partial t} \frac{\partial t}{\partial y}; \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial r} \frac{\partial r}{\partial z} + \frac{\partial}{\partial t} \frac{\partial t}{\partial z}; \quad \text{Where, } \frac{\partial t}{\partial x} = \frac{\partial t}{\partial y} = \frac{\partial t}{\partial z} = 0$$

So, the operator  $\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$  can be expressed as:

$$\nabla = \frac{\partial}{\partial r} \frac{\partial r}{\partial x}\mathbf{i} + \frac{\partial}{\partial r} \frac{\partial r}{\partial y}\mathbf{j} + \frac{\partial}{\partial r} \frac{\partial r}{\partial z}\mathbf{k} = \frac{\partial}{\partial r} \frac{x}{r}\mathbf{i} + \frac{\partial}{\partial r} \frac{y}{r}\mathbf{j} + \frac{\partial}{\partial r} \frac{z}{r}\mathbf{k} = \frac{\partial}{\partial r} \frac{\mathbf{r}}{r} \quad \Rightarrow \quad \nabla \bullet \nabla = \nabla^2 = \frac{\partial^2}{\partial r^2}$$

Thus, Wave Equation can be put in a simpler manner, only as function of  $r$  and  $t$ :

$$\frac{\partial^2 \varepsilon}{\partial r^2} - \frac{1}{c^2} \cdot \frac{\partial^2 \varepsilon}{\partial t^2} = 0; \quad \text{For: } \left\{ \begin{array}{l} \mathbf{r}' = \frac{\mathbf{r} - v \cdot \mathbf{t}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \frac{\partial r'}{\partial t} = \frac{-v}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \frac{\partial r'}{\partial r} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \\ \mathbf{t}' = \frac{\mathbf{t} - \frac{v}{c^2} \cdot \mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \frac{\partial t'}{\partial t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad \frac{\partial t'}{\partial r} = \frac{-\frac{v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}; \end{array} \right.$$

By applying Chain rule for partial derivation, respect to variables  $r, t$ :

$$\frac{\partial \varepsilon}{\partial r} = \frac{\partial \varepsilon}{\partial r'} \frac{\partial r'}{\partial r} + \frac{\partial \varepsilon}{\partial t'} \frac{\partial t'}{\partial r} \qquad \frac{\partial \varepsilon}{\partial t} = \frac{\partial \varepsilon}{\partial r'} \frac{\partial r'}{\partial t} + \frac{\partial \varepsilon}{\partial t'} \frac{\partial t'}{\partial t}$$

Substituting values previously obtained:

$$\frac{\partial \varepsilon}{\partial r} = \frac{\partial \varepsilon}{\partial r'} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{\partial \varepsilon}{\partial t'} \frac{-\frac{v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}; \qquad \frac{\partial \varepsilon}{\partial t} = \frac{\partial \varepsilon}{\partial r'} \frac{-v}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{\partial \varepsilon}{\partial t'} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}};$$

By differentiating again, in order to form all required quadratics components of Wave Equation:

$$\frac{\partial^2 \varepsilon}{\partial r^2} = \frac{1}{\left(1 - \frac{v^2}{c^2}\right)} \left( \frac{\partial^2 \varepsilon}{\partial r'^2} + \frac{v^2}{c^4} \frac{\partial^2 \varepsilon}{\partial t'^2} - 2 \frac{v}{c^2} \frac{\partial^2 \varepsilon}{\partial r' \partial t'} \right)$$

$$\frac{\partial^2 \varepsilon}{\partial t^2} = \frac{1}{\left(1 - \frac{v^2}{c^2}\right)} \left( v^2 \frac{\partial^2 \varepsilon}{\partial r'^2} + \frac{\partial^2 \varepsilon}{\partial t'^2} - 2 \cdot v \cdot \frac{\partial^2 \varepsilon}{\partial r' \partial t'} \right)$$

And substituting these obtained expressions in Wave Equation, we finally have:

$$\begin{aligned} \frac{\partial^2 \varepsilon}{\partial r^2} - \frac{1}{c^2} \cdot \frac{\partial^2 \varepsilon}{\partial t^2} &= \\ &= \frac{1}{\left(1 - \frac{v^2}{c^2}\right)} \left( \frac{\partial^2 \varepsilon}{\partial r'^2} + \frac{v^2}{c^4} \frac{\partial^2 \varepsilon}{\partial t'^2} - 2 \frac{v}{c^2} \frac{\partial^2 \varepsilon}{\partial r' \partial t'} \right) - \frac{1}{c^2} \cdot \frac{1}{\left(1 - \frac{v^2}{c^2}\right)} \left( v^2 \frac{\partial^2 \varepsilon}{\partial r'^2} + \frac{\partial^2 \varepsilon}{\partial t'^2} - 2 \cdot v \cdot \frac{\partial^2 \varepsilon}{\partial r' \partial t'} \right) \end{aligned}$$

$\frac{\partial^2 \varepsilon}{\partial r^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon}{\partial t^2} = \frac{\partial^2 \varepsilon}{\partial r'^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon}{\partial t'^2}$
---

In this way, it is shown that TVL are consistent with Wave Equation (and with Maxwell Equations). So, Wave Equation under TVL has the same presentation for one and another observer, independent of the path of the moving observer and also independent of the light pulse direction, meeting in such a way Einstein relativistic postulates and being consistent with Maxwell Equations.

The problem that LT have, according to our development, is precisely the following assumptions:

$y' = y$  and  $z' = z$  that originates  $\frac{\partial^2 \varepsilon_x}{\partial y^2} = \frac{\partial^2 \varepsilon_x}{\partial y'^2}$  and  $\frac{\partial^2 \varepsilon_x}{\partial z^2} = \frac{\partial^2 \varepsilon_x}{\partial z'^2}$ . They don't allow a vectorial

treatment through variables  $r, t$ . For instance in two dimensions, according to Lorentz, the displacement of light measured by fixed observer at O, is  $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$ , and the corresponding measurement done by moving one at O' is:

$$\mathbf{r}' = x'\mathbf{i} + y'\mathbf{j} = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}\mathbf{i} + y\mathbf{j} = \frac{x}{\sqrt{1 - \frac{v^2}{c^2}}}\mathbf{i} + y\mathbf{j} - \frac{-vt}{\sqrt{1 - \frac{v^2}{c^2}}}\mathbf{i}.$$

By observing carefully this last equation, we conclude that it is not possible to obtain an explicit expression of  $\mathbf{r}'$  as function of  $\mathbf{r}$  and  $\mathbf{t}$ . Thus, it can not be possible to obtain an expression for  $\frac{\partial r'}{\partial r}$ , neither for  $\frac{\partial t'}{\partial r}$ . In these circumstances we can not continue with the procedure of constructing

the vectorial version of the original LT. It can be shown that LT really are not invariant to the Wave equation. Although in some books appears a "demonstration" of the consistency of the LT with Wave Equation, this is not quite general, this is actually a demonstration that is valid only for the particular case of one dimension: the X axis, in where the assumptions cancel out. The chosen example for such demonstration is always presented without any variation: An observer at the origin of the moving system O', which moves on the X axis and a light pulse is sent to the "space" with the usual assumptions. For instance, if this presentation is changed, by establishing that the pulse of light is going parallel to the Z axis, maintaining the moving observer on the X axis, the "demonstration" fails. For showing this, we will work out a known example taken from basic electromagnetic theory:

- A) Let an electromagnetic plane wave move on Z axis at light speed,  $z = ct$ , such that the electric field on Y axis,  $\varepsilon_y = \varepsilon_o \cdot \sin k \cdot (z - ct)$ , depends only on the Z coordinate and time. So, field characteristics will be:  $\varepsilon_x = 0$ ;  $\varepsilon_y = \varepsilon_y(z, t)$ ;  $\varepsilon_z = 0$ ;  $x = y = 0$ . Suppose that the system O' is moving along the X axis at a velocity  $v$  and let's assume,  $z' = z$ , in order to be under the same premises of LT.

The relationships that hold for this case, according to LT, are:

$$x' = \frac{-vt}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad x = 0; \quad y' = y = 0; \quad z' = z; \quad t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \frac{\partial x'}{\partial t} = \frac{-v}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad \frac{\partial t'}{\partial t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad \frac{\partial t'}{\partial x} = 0$$

With these premises, we can write:  $\frac{\partial \varepsilon_y}{\partial x} = \frac{\partial \varepsilon_y}{\partial y} = 0$ , and similarly,  $\frac{\partial y'}{\partial t} = 0$ ;  $\frac{\partial t'}{\partial z} = 0$ ; Given that time

is not an explicit variable in the expression of  $z'$ , then  $\frac{\partial z'}{\partial t} = 0$ ; and because  $z' = z$ , then:  $\frac{\partial z'}{\partial z} = 1$ .

In this way, all the equations corresponding to Wave Equation in function of the coordinate components are reduced to:

$$\frac{\partial^2 \varepsilon_y}{\partial x^2} + \frac{\partial^2 \varepsilon_y}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon_y}{\partial t^2} = 0 \quad \Rightarrow \quad \frac{\partial^2 \varepsilon_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon_y}{\partial t^2} = 0$$

Let's try to build the Wave Equation under prime variables. By using the Chain rule we will form components with the prime variables:

$$\begin{aligned} \frac{\partial \varepsilon_y}{\partial z} &= \frac{\partial \varepsilon_y}{\partial x'} \frac{\partial x'}{\partial z} + \frac{\partial \varepsilon_y}{\partial y'} \frac{\partial y'}{\partial z} + \frac{\partial \varepsilon_y}{\partial z'} \frac{\partial z'}{\partial z} + \frac{\partial \varepsilon_y}{\partial t'} \frac{\partial t'}{\partial z} = \frac{\partial \varepsilon_y}{\partial z'} \frac{\partial z'}{\partial z} = \frac{\partial \varepsilon_y}{\partial z'} \Rightarrow \frac{\partial^2 \varepsilon_y}{\partial z^2} = \frac{\partial^2 \varepsilon_y}{\partial z'^2} \\ \frac{\partial \varepsilon_y}{\partial t} &= \frac{\partial \varepsilon_y}{\partial x'} \frac{\partial x'}{\partial t} + \frac{\partial \varepsilon_y}{\partial y'} \frac{\partial y'}{\partial t} + \frac{\partial \varepsilon_y}{\partial z'} \frac{\partial z'}{\partial t} + \frac{\partial \varepsilon_y}{\partial t'} \frac{\partial t'}{\partial t} = \frac{\partial \varepsilon_y}{\partial x'} \frac{\partial x'}{\partial t} + \frac{\partial \varepsilon_y}{\partial t'} \frac{\partial t'}{\partial t} \\ \frac{\partial^2 \varepsilon_y}{\partial t^2} &= \frac{\partial^2 \varepsilon_y}{\partial x'^2} \frac{\partial x'^2}{\partial t^2} + \frac{\partial^2 \varepsilon_y}{\partial t'^2} \frac{\partial t'^2}{\partial t^2} - 2 \left[ \frac{\partial^2 \varepsilon_y}{\partial x' \partial t'} \frac{\partial x' \partial t'}{\partial t^2} \right] \end{aligned}$$

Substituting by their values we obtain:

$$\frac{\partial^2 \varepsilon_y}{\partial t^2} = \frac{1}{\left(1 - \frac{v^2}{c^2}\right)} \left( v^2 \frac{\partial^2 \varepsilon_y}{\partial x'^2} + \frac{\partial^2 \varepsilon_y}{\partial t'^2} - 2.v. \frac{\partial^2 \varepsilon_y}{\partial x' \partial t'} \right)$$

Introducing these results, it is obtained a different

presentation of the Wave Equation for the prime variables: contrary to what is expected:

$$\boxed{\frac{\partial^2 \varepsilon_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon_y}{\partial t^2} = \frac{\partial^2 \varepsilon_y}{\partial z'^2} - \frac{1}{c^2} \frac{1}{\left(1 - \frac{v^2}{c^2}\right)} \left( v^2 \frac{\partial^2 \varepsilon_y}{\partial x'^2} + \frac{\partial^2 \varepsilon_y}{\partial t'^2} - 2.v. \frac{\partial^2 \varepsilon_y}{\partial x' \partial t'} \right)}$$

This result shows how the original LT **are not really consistent** with the Maxwell Equations, because it does not preserve the structure of Wave Equation.

B) Let's do the same job but through the VLT. Expressing the movement of O' and that of the light pulse in a vectorial form, and remembering that components of vector time measured by O are given by the movement of O', we get:

$$\alpha = \beta = x = y = t_y = t_z = 0; \Rightarrow \mathbf{t} = t\mathbf{i}; \quad \mathbf{r} = z\mathbf{k} \Rightarrow z = r; \quad \text{By applying: } \left\{ \begin{array}{l} \mathbf{t}' = k.(\mathbf{t} - \frac{v}{c^2}.\mathbf{r}) \\ \mathbf{r}' = k.(\mathbf{r} - v.\mathbf{t}) \end{array} \right\}$$

$$\mathbf{t}' = \frac{t\mathbf{i} - \frac{v}{c^2}.r\mathbf{k}}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad \mathbf{r}' = \frac{r\mathbf{k} - v.t\mathbf{i}}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \left\{ \begin{array}{l} \frac{\partial t'}{\partial t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad \frac{\partial t'}{\partial r} = \frac{-\frac{v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \\ \frac{\partial r'}{\partial r} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad \frac{\partial r'}{\partial t} = \frac{-v}{\sqrt{1 - \frac{v^2}{c^2}}} \end{array} \right\}$$

Wave Equation, in function of  $r, t$  had become:  $\frac{\partial^2 \varepsilon_y}{\partial r^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon_y}{\partial t^2} = 0$ . Operating as before, and substituting values, primed Wave Equation is consistently obtained:

$$\frac{\partial^2 \varepsilon}{\partial r^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon}{\partial t^2} = \left[ \frac{\partial^2 \varepsilon}{\partial r'^2} \frac{\partial r'^2}{\partial r^2} + \frac{\partial^2 \varepsilon}{\partial t'^2} \frac{\partial t'^2}{\partial r^2} - 2. \frac{\partial^2 \varepsilon}{\partial r'.\partial t'} \frac{\partial r'.\partial t'}{\partial r^2} \right] - \frac{1}{c^2} \left[ \frac{\partial^2 \varepsilon}{\partial r'^2} \frac{\partial r'^2}{\partial t^2} + \frac{\partial^2 \varepsilon}{\partial t'^2} \frac{\partial t'^2}{\partial t^2} - 2. \frac{\partial^2 \varepsilon}{\partial r'.\partial t'} \frac{\partial r'.\partial t'}{\partial t^2} \right]$$

$$\frac{\partial^2 \varepsilon}{\partial r^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon}{\partial t^2} = \frac{\partial^2 \varepsilon}{\partial r'^2} - \frac{1}{c^2} \frac{\partial^2 \varepsilon}{\partial t'^2}$$

As so it was expected. This also means that VLT are truly consistent with Wave Equation and in general with Maxwell Equations.