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Mass in Vectorial Relativity

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ABSTRACT: In this review, following Franco's original work it is obtained a unique Local Lorentz Transformation (LLT) of mass, which differs from known Einstein's transverse mass definition. After this simple and unique transformation of mass the rest of dynamical LLTs are readily derived. The emphasis in qualify as unique these transformations is because Lorentz Transformations' wrong assumptions make us to be used to work with longitudinal and transverse (parallel or perpendicular components to the body's motion) transformations of physical magnitudes within the frame of the Special Theory of Relativity (SRT).

KEYWORDS: Special Relativity, Relativistic Mass, Relativistic Energy, Relativistic Momentum and in general Relativistic Dynamical Physical Magnitudes.

I FORCE IN VECTORIAL RELATIVITY

Let's try to obtain a dynamical transformation for Force, based on already known LLT of magnitudes.

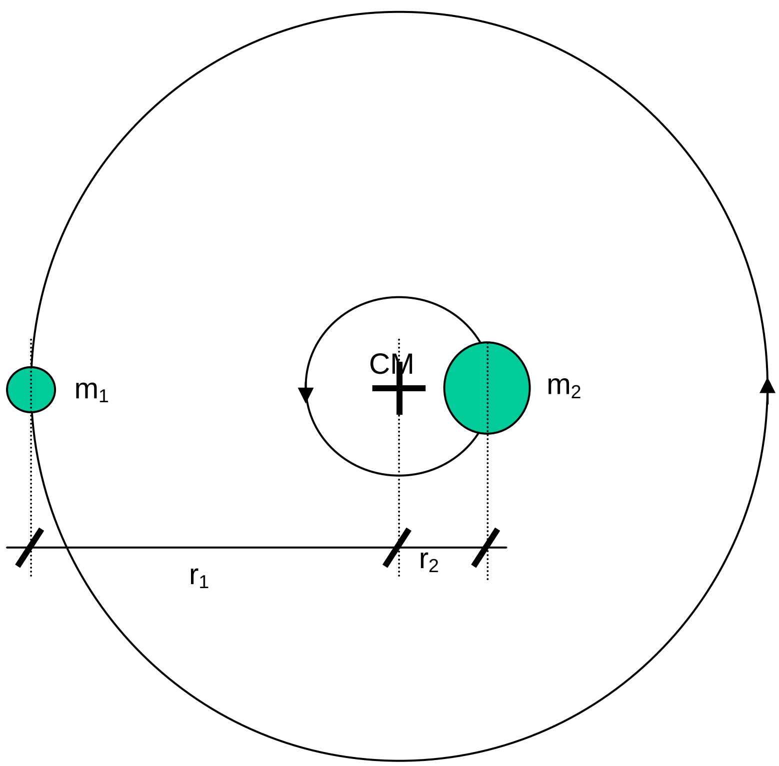

Fig. 1 Two masses rotating around a fixed center C

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Let's suppose two masses, m_1 and m_2, rotating circularly around the center of mass C of the system of the two masses, see Fig. 1. They additionally must move such that their centers of masses are always on a line passing through the center of mass of the system of the two masses, in order to ensure they move at the same angular velocity ω.

Because we have made the masses to describe circular paths, it will allow us to take over gravitational forces in the analysis, i.e., only centrifugal forces will be considered. Let's suppose a Hercules, located at the center of mass C, fixed, sustaining each mass through strong cords with each arm. Let there be three observers: Hercules at C, as a fixed reference; observer 1, first moving reference on mass m_1 at a cord-distance r_1 from C, and observer 2, second moving reference on mass m_2 at a cord-distance r_2 from C.

1) As a first conclusion, for Hercules to be in equilibrium, he must measure equal and opposite tensions in each arm. Thus: $m_1.\omega^2.r_1 = m_2.\omega^2.r_2$.

2) The tension T_1 exerted at one of Hercules' arm by cord r_1, measured by observer 1 on m_1, by taking point C as reference, will be $m'_1.\omega'^2.r'_1$, and tension T_2 exerted at Hercules' other arm by the cord r_2, measured by observer 2, by taking also the point C as reference, on m_2, will be $m''_2.\omega''^2.r''_2$. Let's *assume* that tensions T_1 and T_2 are equal, in order to maintain, as before, Hercules in equilibrium, which could also mean that Force would be invariant under LLT measurements. The values of physical magnitudes of moving masses involved in both tensions transform within the equations according to known LLTs, with respect to what is measured by Hercules, the fixed observer in the following manner (except for masses, whose transformation is unknown):

$$m'_1.\omega'^2.r'_1 = m'_1.\frac{\omega^2}{\left(1-\frac{v_1^2}{c^2}\right)}.\frac{r_1}{\sqrt{1-\frac{v_1^2}{c^2}}} \equiv m''_2.\omega''^2.r''_2 = m''_2.\frac{\omega^2}{\left(1-\frac{v_2^2}{c^2}\right)}.\frac{r_2}{\sqrt{1-\frac{v_2^2}{c^2}}}$$

The only way for this relationship to always be consistent for any values of v_1 and v_2 is that masses have the following LLT:

$$m'_1 = \left(1-\frac{v_1^2}{c^2}\right)^{\frac{3}{2}}.m_1 \quad \text{and} \quad m''_2 = \left(1-\frac{v_2^2}{c^2}\right)^{\frac{3}{2}}.m_2 \qquad (1)$$

In this way Lorentz factors cancel out and this would imply that: $m_1.\omega^2.r_1 = m_2.\omega^2.r_2$, But, as this equality was previously correctly concluded in 1), then our assumption that tensions T_1 and T_2 is correct. This can be seen in another way. For maintaining Hercules in equilibrium (first conclusion), then tensions T_1 and T_2 must be equal. Thus, these results lead to that they are implied to each other.

, i.e.,

$$T_1 = m'_1.\omega'^2.r'_1 \equiv m_1.\omega^2.r_1 \equiv m_2.\omega^2.r_2 \equiv m''_2.\omega''^2.r''_2 = T_2.$$

Let's discuss in a deep way this equation. When observer 1 on m_1 (remember that he is fixed with respect to this mass, although the whole is a moving system) measures his mass, he measures m'_1, which is, for him, the rest mass, $m'_1 = M_{01}$. The same applies for the other observer 2 measuring the mass where he is on: $m''_2 = M_{02}$. So, given that through this special case of circular movement we have obtained the Lorentz factors for such masses in (1), and because the LLT of a magnitude always has the same structure, we can conclude, from equation (1), with the following strong statement: **In general, an inertial mass in movement at a velocity v is related to its rest mass in the following manner**:

$$m = \frac{M_0}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \tag{2}$$

This definition differs from the well-known Einstein's mass definition: $m = \frac{M_0}{\sqrt{1-\frac{v^2}{c^2}}}$. **In regards with this point, it's worth mentioning that Einstein also obtained equation (2) in his remarkable paper of 1905. He called this mass "longitudinal mass" [2], but later he discarded it from his work.**

Continuing the analysis by another route to obtain the relationship between mass and velocity, the following one is a more general way to arrive at the same result. For instance, let's consider a pair of mass, for instance the Sun and Earth as if they were the only bodies of the inertial solar system. Let's consider that as if the sun was the fixed reference (center of Sun is almost the center of mass of this system), and Earth moving around the Sun. Thus, Angular Momentum of Earth under LLT, measured by an observer from the Sun, is $m.r^2.\omega$ and its value must be constant, because there are no more forces acting around, and conservation of angular momentum holds. The "same" Angular Momentum of Earth, which moves following an elliptical path with variable velocity, measured by another observer, on Earth, taking Sun as his reference for measurements, is $m'.r'^2.\omega'$, which must also be constant, because the laws of nature are the same in any system of coordinates, becomes:

$$m'.r'^2.\omega' = m'.\frac{r^2}{\left(1-\frac{v^2}{c^2}\right)}.\frac{\omega}{\sqrt{1-\frac{v^2}{c^2}}} = CONSTANT \tag{3}$$

Let's focus our attention on the explicit transformation of the elements involved within this last expression of angular momentum except for the earth mass, whose transformation is still considered unknown. By carefully observing the equation (3), we conclude that the only way for this expression being always constant, for any value of the variable v present in Lorentz factors in the denominator, is that the transformation for $m' = M_0$, cancels out the effect of such factors. For instance:

$$m' = M_0 = \left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}.m \quad \Rightarrow \quad m = \frac{M_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \tag{4}$$

This is the same result previously obtained in equation (2). This also means that Angular Momentum is invariant under LLT (and also the force). Given that Local Lorentz factors influence magnitude uniformly in all dimensions: We don't have different LLT's for the same magnitude, contrasting to which is found in the Special Theory of Relativity (remember longitudinal or transversal expressions of mass, fields, etc).

II OTHER DYNAMICAL LLTs IN VECTORIAL RELATIVITY

Given that LLT of mass is already known, according to result in (4), let's obtain other dynamical LLT.

Linear Momentum: $$\mathbf{p}' = m'.\mathbf{v}' = m.\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}.\frac{\mathbf{v}}{\left(1 - \frac{v^2}{c^2}\right)} \quad \Rightarrow \quad \mathbf{p}' = \sqrt{1 - \frac{v^2}{c^2}}.\mathbf{p} \tag{5}$$

Observe this result: Linear Momentum is not invariant, as SRT states.

Angular Momentum: $$\mathbf{L}' = \mathbf{r}' \times \mathbf{p}' = \frac{\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} \times \sqrt{1 - \frac{v^2}{c^2}}.\mathbf{p} \Rightarrow \mathbf{L}' = \mathbf{L} \text{ (Invariant)} \tag{6}$$

Force: $$\mathbf{F}' = \frac{d\mathbf{p}'}{dt'} = \frac{\sqrt{1 - \frac{v^2}{c^2}}.d\mathbf{p}}{\sqrt{1 - \frac{v^2}{c^2}}.dt} \quad \Rightarrow \quad \mathbf{F}' = \mathbf{F} \quad \text{(Invariant, as expected)} \tag{7}$$

Kinetic Energy: $$dE' = \mathbf{F}'.d\mathbf{r}' = \mathbf{F}.\frac{d\mathbf{r}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \Rightarrow \quad dE' = \frac{dE}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{8}$$

Electromagnetic or Lorentz Force: It must be invariant, because the magnitude of any force is invariant, see equation (7) and development of equation (2).

$$\mathbf{F}' = q'.(\vec{\Xi}' + \mathbf{v}' \times \mathbf{B}')$$

Let's discuss this relationship. Electric Charge q seems not to be influenced by the velocity. Let's assume that it is invariant under LLT. Thus, in order to preserve the invariance of Force, Electric Field $\vec{\Xi}$ and the product $\mathbf{v} \times \mathbf{B}$ must be invariant. ***If this assumption is false, for sure a contradiction will arise later on***. A good property of scaling factors in LLT is that they behave as if they had magnitude, allowing dimensional analysis based on characteristic Lorentz scaling factors of physical magnitudes. From the assumed LLT invariance of q, then $\mathbf{v}' \times \mathbf{B}'$ is also invariant:

Magnetic Field Density: Knowing that $\mathbf{v}' = \dfrac{\mathbf{v}}{\left(1 - \dfrac{v^2}{c^2}\right)} \Rightarrow \mathbf{B}' = \mathbf{B}.\left(\mathbf{1} - \dfrac{v^2}{c^2}\right)$ (9)

Electric Field: $\vec{\Xi}' = \vec{\Xi}$ (Invariant) (10)

Electric Charge: $q' = q$ (Invariant) (11)

Electric Potential: $dV' = \vec{\Xi}'.d\mathbf{r}' = \vec{\Xi}.\dfrac{d\mathbf{r}}{\sqrt{\mathbf{1} - \dfrac{v^2}{c^2}}} \Rightarrow dV' = \dfrac{dV}{\sqrt{\mathbf{1} - \dfrac{v^2}{c^2}}}$ (12)

Let's check the assumption for Electric Charge and its effects. Let's obtain the expression for the Electric Energy. It should lead to the already obtained expression (8) for energy. In fact:

$$dE' = q'.dV' = q.\frac{dV}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow dE' = \frac{dE}{\sqrt{1 - \frac{v^2}{c^2}}}$$ [See equation (8)]

Another check: An electric charge contained in a mass m located in an uniform magnetic field, which moves describing a circular path, should lead to the LLT of angular velocity, which is already known from previous section.

$$\boldsymbol{\omega}' = -q'.\frac{\mathbf{B}'}{m'} = -q.\frac{\mathbf{B}.\left(\mathbf{1} - \frac{v^2}{c^2}\right)}{m.\left(\mathbf{1} - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} = \frac{-q.\frac{\mathbf{B}}{\mathbf{m}}}{\sqrt{\mathbf{1} - \frac{v^2}{c^2}}} \Rightarrow \boldsymbol{\omega}' = \frac{\boldsymbol{\omega}}{\sqrt{\mathbf{1} - \frac{v^2}{c^2}}}$$ (Check previous Review on LLT)

As it is has seen, our assumption for charge is consistent. Furthermore, this control ratifies mass transformation. Let's continue checking:

The Magnetic Field Density B on a point at a distance R from a current $I=\frac{dq}{dt}$, given by $B=\frac{\mu.I}{2\pi.R} \Rightarrow \mu=\frac{2\pi.R.B}{I}$, leads to obtain the transformation of the

Magnetic Permeability: $$\mu'=\frac{2\pi.\frac{R}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}.B.\left(1-\frac{v^2}{c^2}\right)}{\frac{dq}{dt.\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}} \Rightarrow \mu'=\mu.\left(1-\frac{v^2}{c^2}\right) \tag{13}$$

Similarly, Electric Permittivity, ε, can be obtained from Gauss Law, and using dimensional analysis:

$$\oint_{S'}\vec{\Xi}'.d\mathbf{S}'=\frac{q'}{\varepsilon'} \quad\Rightarrow\quad \oint_{S}\vec{\Xi}.\frac{d\mathbf{S}}{\left(1-\frac{v^2}{c^2}\right)}=\frac{q}{\varepsilon'} \quad\Rightarrow\quad \varepsilon'=\varepsilon.\left(1-\frac{v^2}{c^2}\right) \tag{14}$$

Electric Displacement: $$\mathbf{D}'=\varepsilon'.\vec{\Xi}'=\varepsilon.\left(1-\frac{v^2}{c^2}\right).\vec{\Xi} \quad\Rightarrow\quad \mathbf{D}'=\mathbf{D}.\left(1-\frac{v^2}{c^2}\right) \tag{15}$$

Current Density: $$\mathbf{J}'=\frac{\frac{dq'}{dt'}}{S'}=\frac{\frac{dq}{dt.\sqrt{1-\frac{v^2}{c^2}}}}{\frac{S}{\left(1-\frac{v^2}{c^2}\right)}} \quad\Rightarrow\quad \mathbf{J}'=\mathbf{J}.\sqrt{1-\frac{v^2}{c^2}} \tag{16}$$

Magnetic Field: $$\mathbf{H}'=\frac{\mathbf{B}'}{\mu'}=\frac{\mathbf{B}.\left(1-\frac{v^2}{c^2}\right)}{\mu.\left(1-\frac{v^2}{c^2}\right)} \quad\Rightarrow\quad \mathbf{H}'=\mathbf{H} \quad \text{(Invariant)} \tag{17}$$

Magnetic Flux: $$\phi'=\oint_{S'}\mathbf{B}'.d\mathbf{S}'=\oint_{S'}\mathbf{B}.\left(1-\frac{v^2}{c^2}\right).\frac{d\mathbf{S}}{\left(1-\frac{v^2}{c^2}\right)} \Rightarrow \phi'=\phi \quad \text{(Invariant)} \tag{18}$$

Checking: $\frac{\partial \vec{\Xi}'}{\partial r'} = -\frac{\partial \mathbf{B}'}{\partial t'} \Rightarrow \frac{\partial \vec{\Xi}}{\frac{\partial r}{\sqrt{1-\frac{v^2}{c^2}}}} = -\frac{\partial \mathbf{B}.\left(1-\frac{v^2}{c^2}\right)}{\partial t.\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \frac{\partial \vec{\Xi}}{\partial r} = -\frac{\partial \mathbf{B}}{\partial t}$ [Checked]

$$-\frac{\partial \mathbf{B}'}{\partial r'} = -\mu'.\varepsilon'.\frac{\partial \vec{\Xi}'}{\partial t'} \Rightarrow \frac{\partial \mathbf{B}.\left(1-\frac{v^2}{c^2}\right)}{\frac{\partial r}{\sqrt{1-\frac{v^2}{c^2}}}} = -\mu.\varepsilon.\left(1-\frac{v^2}{c^2}\right)^2 \frac{\partial \vec{\Xi}}{\partial t.\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow -\frac{\partial \mathbf{B}}{\partial r} = -\mu.\varepsilon.\frac{\partial \vec{\Xi}}{\partial t}$$

Also checked! These results show that the relation between electric and magnetic fields holds in any reference system, as it was expected.

It can be shown that Maxwell Equations hold in any reference system under LLT. In fact, by taking into account that:

$$\nabla' = \frac{\partial}{\partial r'} = \frac{\partial}{\frac{\partial r}{\sqrt{1-\frac{v^2}{c^2}}}} \Rightarrow \nabla' = \sqrt{1-\frac{v^2}{c^2}}.\nabla : \qquad (19)$$

1) $\nabla'\times\vec{\Xi}' = -\frac{\partial \mathbf{B}'}{\partial t'} \Rightarrow \sqrt{1-\frac{v^2}{c^2}}\nabla\times\vec{\Xi} = -\frac{\partial \mathbf{B}\left(1-\frac{v^2}{c^2}\right)}{\partial t.\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \nabla\times\vec{\Xi} = -\frac{\partial \mathbf{B}}{\partial t}$

2) $\nabla'\times\mathbf{H}' = \frac{\partial \mathbf{D}'}{\partial t'} + \mathbf{J}' \Rightarrow \sqrt{1-\frac{v^2}{c^2}}\nabla\times\mathbf{H} = \frac{\partial \mathbf{D}.\left(1-\frac{v^2}{c^2}\right)}{\partial t.\sqrt{1-\frac{v^2}{c^2}}} + \mathbf{J}.\sqrt{1-\frac{v^2}{c^2}} \Rightarrow \nabla\times\mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$

3) $\nabla'\bullet\mathbf{D}' = \rho' = \frac{q'}{V'} \Rightarrow \sqrt{1-\frac{v^2}{c^2}}\nabla\bullet\mathbf{D}.\left(1-\frac{v^2}{c^2}\right) = \frac{q}{\frac{V}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}}} \Rightarrow \nabla\bullet\mathbf{D} = \rho$

4) $\nabla'\bullet\mathbf{B}'=\mathbf{0} \quad\Rightarrow\quad \nabla\bullet\mathbf{B}.\left(1-\frac{v^2}{c^2}\right)=\mathbf{0} \quad\Rightarrow\quad \nabla\bullet\mathbf{B}=\mathbf{0}$

Pointing Theorem. For $P'=\frac{dE'}{dt'}=\frac{\frac{dE}{\sqrt{1-\frac{v^2}{c^2}}}}{dt.\sqrt{1-\frac{v^2}{c^2}}}=\frac{P}{\left(1-\frac{v^2}{c^2}\right)}$ (20)

$$P'=\mathrm{Re}\frac{1}{2}\oint\vec{\boldsymbol{\varepsilon}}'\times\mathbf{H}'\cdot d\mathbf{S}' \Rightarrow \frac{P}{\left(1-\frac{v^2}{c^2}\right)}=\mathrm{Re}\frac{1}{2}\oint\vec{\boldsymbol{\varepsilon}}\times\mathbf{H}\cdot\frac{d\mathbf{S}}{\left(1-\frac{v^2}{c^2}\right)} \Rightarrow P=\mathrm{Re}\frac{1}{2}\oint\vec{\boldsymbol{\varepsilon}}\times\mathbf{H}\cdot d\mathbf{S}$$

Observe the dimensional analysis' consistency of LLTs for each magnitude. All these consistent controls seem to confirm the correctness of the LLT approach.

III TRANSFORMATIONS AND NATURAL LAWS

It is necessary to remark that Lorentz factors are only scaling factors between measurements done by two observers, no matter whether they are differentials or integrals. For instance, the following transformations become:

If $\mathbf{p}'=\sqrt{1-\frac{v^2}{c^2}}.\mathbf{p}$, then, $d\mathbf{p}'=\sqrt{1-\frac{v^2}{c^2}}.d\mathbf{p}$, or $\iiint d^3\mathbf{p}'=\iiint\sqrt{1-\frac{v^2}{c^2}}.d^3\mathbf{p}$

However, a different thing is the following situation: the contraction suffered by a bar going through the space with velocity v, from a known length L_0, to L, measured by one observer inside his own system (second observer does not exist), according to the law $L=L_0.\sqrt{1-\frac{v^2}{c^2}}$. This case is not a scaling one in the sense of the LLT, but a property of the bar whose **known length** depends on its velocity through the space for a stationary observer. For velocity v, variable, L is also variable. For this case the differential dL becomes, $dL=L_0.\frac{v.dv}{\left(1-\frac{v^2}{c^2}\right)}$.

The same consideration must be made for a mass m that crosses the space with velocity v, with known rest mass m_0. The expressions for the variable m, and its differential, depending on its velocity v, are:

$$m = \frac{M_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \quad \Rightarrow \quad dm = 3.M_0.c^3.\frac{v.dv}{\left(c^2 - v^2\right)^{\frac{5}{2}}} = 3.\frac{M_0.c^3}{\left(c^2 - v^2\right)^{\frac{3}{2}}}.\frac{v.dv}{\left(c^2 - v^2\right)} = 3.m.\frac{v.dv}{\left(c^2 - v^2\right)}$$

As it should be noticed, it is important to be careful with such differences.

Another aspect to be emphasized is that of the vector character of time. This vectorial character is only noticed within the relationship between times measured by two inertial observers, through coordinate transformations, when a generalized configuration is used. Only under such condition, time is mathematically forced to appear as a vector. On the contrary, time measured by one observer in his own coordinate system (second observer does not exist) can behave as we are used to know it: as a scalar, although as it is encountered in the work done by Hongbao Ma, it is possible to express time in a vectorial form [4].

IV CONCLUSIONS

If Einstein's postulates about the constancy of the speed of light and the principle of relativity are correct (in authors' opinion they are); if assumptions within Lorentz Transformations (LT) are wrong as it was shown in Franco's work; if the development of Vectorial Lorentz Transformations (VLT) trying to correct the misconceptions presented in the referred work [5] and in previous Reviews, reveals itself to be the correct approach to the consideration of the speed of light as a universal constant for any inertial observer, and if the Local Lorentz Transformations are the a correct application of those results for obtaining the influence of the speed of light on the physical magnitudes of a body in movement, it will lead to interpret correctly our universe and an to give an enormous contribution to physics to get again its way, soft-transiting its development from Classic to Relativistic and/or to Quantum Physics. In authors' opinion, Einstein's work was intended to go in this sense, but the unsolved contradictions, introduced by Lorentz Transformations (as it was informed in previous Reviews) at the very starting point of his research, probably made Einstein leave in an abrupt manner the Special Theory of Relativity, as in our opinion he did and it happened, to lead back his investigation into a more general development, to which he named the General Theory of Relativity. Experimental validation of Franco's approach against Einstein one, for example, the accuracy of the new definition of mass rigorously obtained in equation (4) of this Review, will probably require complex experiments with known rest masses accelerated at speeds close to that of light in order to establish whether the value of mass is the well-known coined by Einstein (transverse mass) or that given by the equation (4). This task probably could be obtained with the beginning of the new particle accelerator with mass in rest, which is being build at CERN and will be operating at the end of year 2007. See in the "News", in the first part of this Journal, the article **"European new particle accelerator: The Large Hadron Collider (LHC)".**

REFERENCES

[1] H. A. Lorentz. *Electromagnetic Phenomena in a System Moving with any Velocity less than that of Light.* Proc. Acad. Sci. of Amsterdam, **6**, 1904.

[2] Albert Einstein. *Zur Elektrodynamik bewegter Körper,* Annalen der Physic 17, 1905, pp. 891-921. English version. *On the Electrodynamics of Moving Bodies.* http://www.fourmilab.ch/etexts/einstein/specrel/www/

[3] Albert Einstein. *The Meaning of Relativity*, Fifth Edition, MJF Books, New York, 1956. Page 34.

[4] Hongbao Ma. *The Nature of Time and Space.* Nature and Science 1 (1) November 2003. Page 8, section 18.

[5] J A Franco R. *Vectorial Lorentz Transformations.* 2006. EJTP 9 (2006) 35-64..