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Local Lorentz Transformations and Vectorial Relativity

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ABSTRACT: In previous work it was shown that instead of the known Lorentz Transformations (LT) the new obtained Vectorial Lorentz Transformations (VLT) were the transformations that truly respected the postulates of the Principle of Relativity and the consideration of the speed of light as a universal constant. In this review is presented the way how it is possible to obtain practical consequences of the VLT, and its applications to our real life.

KEYWORDS: Special Relativity, Relativistic Mass, Relativistic Energy and Relativistic Momentum.

I LOCAL LORENTZ TRANSFORMATIONS

As it is commonly expressed in relativistic literature, the practical consequences of LT, under conditions of simultaneity of events, or their occurrence at the same location, are those known as length contraction and time dilation, respectively.

In order to establish a sequence of steps to arrive at the Local Lorentz Transformations, let's recall the existent conditions within the analysis of the Vectorial Lorentz Transformations:

a) Two inertial systems with relative movements between them and an observer placed in each system with equipment to measure time and distances, previously calibrated, are considered.

b) Because of the relativity of motion, each observer considers his system as fixed and the other moving: It is impossible to demonstrate which system is moving, as it was shown by Einstein in 1905.

c) When both inertial systems coincide a light pulse is sent to the space and measurements of its trajectory are done by each observer within his inertial system "without knowledge of the other observer located in the other inertial system". Thus, each observer measures from his origin of coordinates a different radio-vector to the point P in the space, previously chosen, where the light pulse arrives at.

d) Each observer, in an independent way must measure that the speed of the light pulse is c .

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Under theses conditions, the manner to obtain a "clean" expression of the length contraction, for example of a bar that at this moment is moving at a speed v, but it is known it has a length L_0, measured at rest, is by doing simultaneous measurements. For example, Let A and B the extremes of the bar moving along the X-axis of fixed system at speed v and let x'_A, x'_B, $x'_B - x'_A = L_0$, the measurements done by the moving observer at O', where the bar is located. So, the fixed observer will be able to do distance measurements on the bar, x_A, x_B, simultaneously ($t_A = t_B = t$), and he will obtain: $x_B - x_A = L$. By establishing the VLT of distances and making suitable operations, follows the referred expression of length contraction:

$$x'_B - x'_A = \frac{(x_B - x_A) + v.(t_B - t_A)}{\sqrt{1 - \frac{v^2}{c^2}}} = L_0 = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \Rightarrow \qquad L = L_0 . \sqrt{1 - \frac{v^2}{c^2}}$$

For obtaining the expression of time dilation, let's have two events A and B, as for example two shots of a weapon occurring at the same place ($x_B = x_A$, $x_B - x_A = 0$) in the fixed system O, at different times (t_A, t_B, $t_B - t_A = t_0$). Measurements done by a moving observer in function of what is measured by the fixed observer is t'_A, t'_B, $t'_B - t'_A = \Delta t'$. For this case time dilation takes the following "clean" expression:

$$t'_B - t'_A = \frac{(t_B - t_A) + \frac{v}{c^2}.(x_B - x_A)}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \Rightarrow \qquad \Delta t' = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

As we have seen in previous examples, to observe a "clean" length contraction it is necessary to have simultaneity of events. Similarly, to observe a "clean" time dilation it needs the events occur at the same place.

When simultaneity or same location are not met complex expressions for length contraction and time dilation arise. Obviously, they come from the fact that we are comparing measurements done from two distinct inertial systems with distinct origins. What about if measurements are done from the same reference? Clearly, we are modifying the original conditions under VLT were developed. Observe that the equalization of the most of conditions is the usual way to compare measurements: Those conditions left free of variation are precisely those to compare. In this case, we only want to compare the measurements of a magnitude done by a fixed observer with those done by another moving one, and what is the amount of their difference, if any.

As a practical result, our final goal is to establish if a variation in the measurement of a magnitude is observed when it is done in movement respect to that at rest. In this sense we will call the transformations *local*, because we are being allowed measuring in our own system of reference the value of a magnitude in movement, knowing its rest value. Namely, given this only reason we will call them Local Lorentz Transformations.

For example, when a physicist conceives that a pulse of light lasts eight and a half minutes coming from the Sun to the Earth, and he receives such image in his eyes, he knows that Sun is not there at that place where he is viewing Sun's image, but 15.300 Km far apart. From this point of view, the physicist is thinking in a way that instantaneously his thought reflects a the real situation of what is in-real-time is really occurring with Sun's motion. In this sense, visual images are never real but thoughts, like the previous one, depicts instantaneously reality.

Thus, we will see next that the situation of simultaneity of events and occurrence at the same place, or the observed contraction of lengths and time dilation in relativity can be obtained by reducing the configuration of two distinct inertial observers measuring a physical magnitude respect to distinct references to the situation of measuring the same physical magnitude from the same reference. What does this mean? As we know, Vectorial Lorentz Transformations are the relationships between measurements of length and time, done by two different moving observers inside their own frame of reference, without knowing each other. When each observer measures the speed of light they do their measurements taking as reference their own origin of coordinates, the origin O in the first case, or for the considered fixed observer, and the origin O' for the considered moving one. Despite this, the result is that the value of speed of light obtained by each observer is the same. Now, let's pose the situation of measurements of physical magnitudes with the different configuration for both observers referred previously, to which we call Local Lorentz Transformations (LLT).

Let's establish that each observer knows about the presence of the other observer, and they agree to measure magnitudes during the "same period of time", by taking as reference the same origin of coordinates, in order to compare their results. These measurements, of course will yield different results if compared with those obtained through Vectorial Lorentz Transformations, which are done by taking different origins of coordinates. We will see also that this new configuration will give us practical relationships that are not dependent on the orientation of the body's movement. Remember the different transformations of magnitudes usually handled in the Special Theory of Relativity: Longitudinal or Transverse Lorentz Transformations.

In order to systematize the ideas and apply them for taking hold of a real comparison of measurements let's establish, for obtaining the referred Local Lorentz Transformations, the following two conventions:

1) From now on, both observers will do their measurements by taking the same point of reference. <u>Let the origin of the fixed observer be this reference</u>. For example, if the fixed observer O measures the radio-vector $\mathbf{r}$ of a projectile sent to the space, see **Fig. 1**, the observer on the moving system O' will measure a similar radio-vector $\mathbf{R}'$ from this same reference of the fixed observer, such that $\mathbf{R}'$ will fulfill $\mathbf{r}' = \mathbf{R}' - \mathbf{R}'_0$, with the definitions of the variables $\mathbf{R}'$ and $\mathbf{R}'_0$ given below:

$$\mathbf{R}' = \frac{\mathbf{r}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\mathbf{r} - v.\mathbf{t}}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{v.\mathbf{t}}{\sqrt{1-\frac{v^2}{c^2}}} = \mathbf{r}' + \mathbf{R}'_0\,; \text{ for } \mathbf{R}'_0 = \frac{v.\mathbf{t}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\mathbf{r}_0}{\sqrt{1-\frac{v^2}{c^2}}} \tag{1}$$

Then, the measurement of moving observer is related to that of the fixed one, only by the scaling factor.

It is important to notice that LLT are also compatible with Maxwell equations.

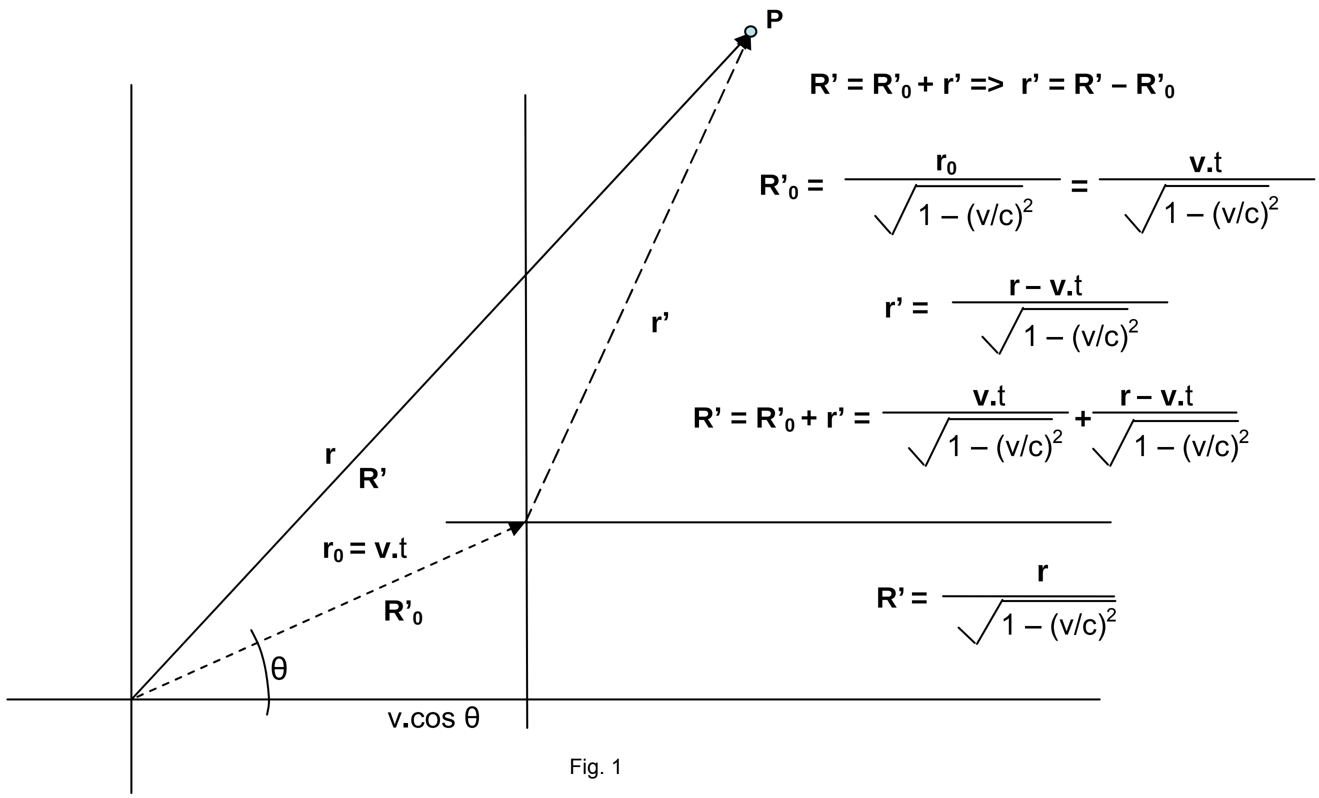

Fig. 1

2) The moving observer O' will not send any pulse of light (or projectile). Thus, he will measure a null displacement of projectile. So, the radio-vector of his moving system, $\mathbf{r}_0$, will be the only measurement completed. Therefore, from the general VLT expressions previously obtained and the conventions applied, the Local Lorentz Transformations (LLT) are obtained as:

$$\mathbf{r}'=0 \Rightarrow \mathbf{r}=v.\mathbf{t}=\mathbf{r_0}\,; \quad \mathbf{t}'=\frac{\mathbf{t}-\dfrac{v}{c^2}.v.\mathbf{t}}{\sqrt{1-\dfrac{v^2}{c^2}}} \Rightarrow \boxed{\mathbf{t}'=\mathbf{t}.\sqrt{1-\frac{v^2}{c^2}}\,; \quad \mathbf{R}'=\frac{\mathbf{r}_0}{\sqrt{1-\dfrac{v^2}{c^2}}}} \tag{2}$$

In sum, the moving and fixed observers are referring their measurements, $\mathbf{R}'$, $\mathbf{t}'$, $\mathbf{t}$, and $\mathbf{r_0}$, of the moving origin O’ respect to the same point of reference: the origin of coordinates of the system O. Given that time is a vector with spatial components, it can be thought as referred to the spatial origin O. As we observe in (2), time and distance vectors are related by a characteristic-scaling factor. The scaling factor, with a value less than unity, in the case of time is a multiplier within each component. For distances, it is a divider. In other words, if any component of a parameter has the same type of factor the contraction or expansion is the same in any direction, for instance, a sphere should expands uniformly in all directions according to the scaling factor depending on its velocity and the speed of light.

However, it is important to point out that the LLT are referred to measurements **relative to the same point of reference: origin O**. This is completely different to what is done for VLT, where each observer does measurements relative to its own reference system. Namely, **the transformations referred to LLT are different to those of VLT.**

<u>With those two conventions in mind</u>, we will not be worried about location coincidence or simultaneity of events. The relations (2) imply that in LLT each physical magnitude observed by an fixed observer, by virtue of its dependency on velocity, in a true way either contract, expand, growth or reduce, with the same scaling factor in all dimensions, independently of their image or how we see them. For example, if on the moving system an observer at O' measures a bar of length L_0, lasting a time t_0 in his measuring, the observer on fixed system at O will measure this length as L and "time t_0" as t. <u>The position</u> of the bar in system O' <u>is not relevant</u>; what is important is that it is at rest for the observer at O' and moving on relative to O. Thus, the relationship between both measurements, according to LLT, will be:

$$L_0 = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}; \quad t_0 = t.\sqrt{1-\frac{v^2}{c^2}} \quad \Rightarrow \quad L = L_0.\sqrt{1-\frac{v^2}{c^2}}; \quad t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} \tag{3}$$

This indicates that an observer in a "stationary" system O measures onto a moving bar at velocity v, a contraction from its original length L_0 to L, no matter which is the position of the bar in the system O', and a time dilation from t_0 to t, as it is shown in equations (3).

Another relevant characteristic is the following one: Lorentz factors in LLT act as scaling factors between measurements done at O and at O', for any magnitude, no matter if this is a differential magnitude or an integral one. In other words, Lorentz factors are simply scaling factors between such measurements.

What is the real meaning of LLT expressed in equations (3)? First of all, **each component is affected by the Lorentz factor in the same way**, namely, contracting lengths. For example, If instead of a bar the observer in the moving system had had a squared bar, whose area, as we know, <u>is the product of two lengths</u>, then the obtained LLT of such area for an observer at the fixed system, would be therefore the product of two contracted lengths (later we will use this LLT characteristic of areas):

$$S' = S_0 = L_0^2 = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}.\frac{L}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{L^2}{1-\frac{v^2}{c^2}} \Rightarrow S_0 = \frac{S}{1-\frac{v^2}{c^2}} \Rightarrow S = S_0.\left(1-\frac{v^2}{c^2}\right) \tag{4}$$

A volume $V' = L_{01}.L_{02}.L_{03}$, measured from O' is related to the volume measured from O, $V = L_1.L_2.L_3$, by the characteristic product of the three contracted lengths given below in (5):

$$V' = V_0 = L_{01}.L_{02}.L_{03} = \frac{L_1}{\sqrt{1-\frac{v^2}{c^2}}}.\frac{L_2}{\sqrt{1-\frac{v^2}{c^2}}}.\frac{L_3}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{L_1 L_2 L_3}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \Rightarrow V_0 = \frac{V}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}}$$

$$\Rightarrow \quad V = V_0.\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}} \qquad (5)$$

The velocity of the origin O' is obtained by differentiating the displacement of O' respect to time and substituting known LLTs. The LLT for velocity becomes:

$$\mathbf{v}' = \frac{d\mathbf{R}'}{dt'} = \frac{\frac{d\mathbf{r_0}}{\sqrt{1-\frac{v^2}{c^2}}}}{dt.\sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{d\mathbf{r_0}}{dt}}{1-\frac{v^2}{c^2}} \qquad \Rightarrow \qquad \mathbf{v}' = \frac{\mathbf{v}}{1-\frac{v^2}{c^2}} \qquad (6)$$

At this moment we realize that velocity of the moving system O', plays two roles: either as a scalar, v, when it is inside the scaling factor, in where both observers see each other moving relative to themselves in the same line **under conditions of VLT**. Or, as a vector $\mathbf{v}'$, measured by the observer at O' into his own frame by taking as reference the origin of the other system O, **under conventions of LLT**. It is important to be conscious with these two different concepts!

After doing this necessary parenthesis, let's continue: LLT for acceleration is obtained in the same manner as in (6):

$$\mathbf{a}' = \frac{d\mathbf{v}'}{dt'} = \frac{\frac{d\mathbf{v}}{1-\frac{v^2}{c^2}}}{dt.\sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{d\mathbf{v}}{dt}}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \qquad \Rightarrow \qquad \mathbf{a}' = \frac{\mathbf{a}}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \qquad (7)$$

It is also necessary to say at this moment that origin O' could also have a motion along an inertial curvilinear path following an inertial movement with variable velocity. For example Earth has an undoubted inertial curvilinear movement around the Sun, and although it accelerates going to perihelion and reduce its speed after perihelion going to aphelion, we don't feel anything, buildings maintain their verticality, equilibrium of any kind is preserved, etc. So, with the found transformations in equations (6) and (7), we would expect to obtain also LLT in Dynamics. Let's do some remarks. As we will demonstrate next, it is possible to apply the Vectorial Lorentz Transformations (VLT), to an inertial system of coordinates with curvilinear movement, with respect to a fixed system, located in a point throughout the curvilinear trajectory of the moving system.

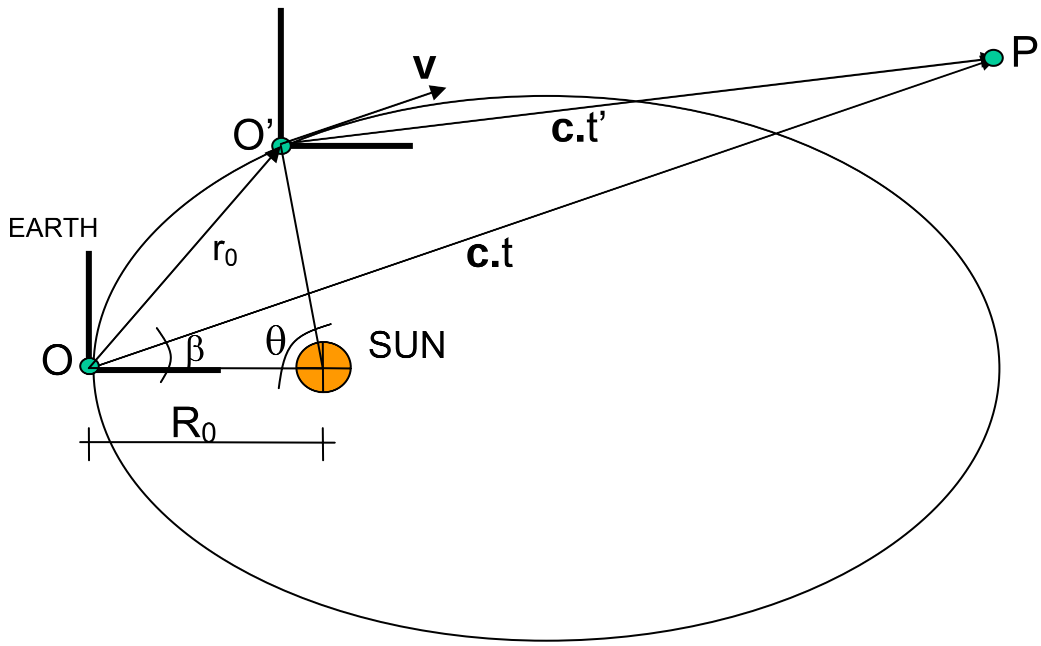
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Fig. 2

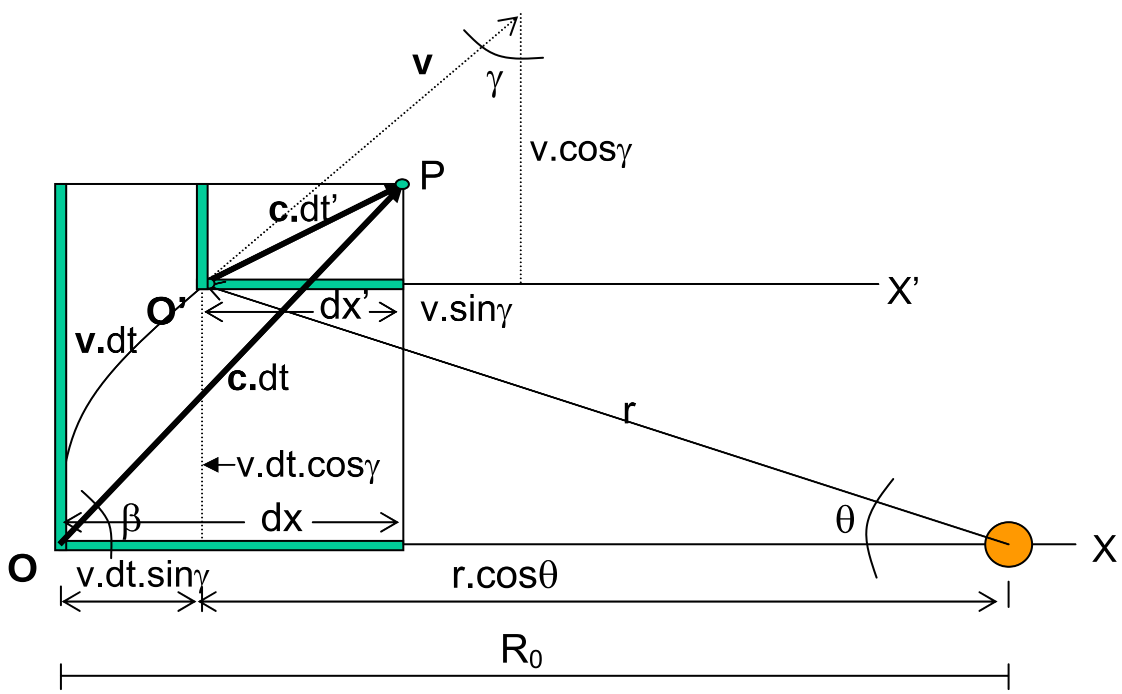
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Fig. 3

We can establish that, inertial systems are not only those with null acceleration, but those where “the sum of acting forces is null”. These include not only those with null acceleration in rectilinear movement, but also those in curvilinear movement with a constant Angular Momentum.

For the movement of Earth around the Sun, the summation of the gravitational force of Sun onto Earth plus the Earth’s centrifugal force gives a null result, reason why the Earth movement is inertial according to since we have defined it. In this way, earth’s movement is neither impeded nor eased by any additional external force. We will try to reproduce this movement in **Fig. 2**, where the first observer is on the moving system, Earth, at O’, and the second observer will be fixed on the elliptic path at the nearest point to the Sun, the perihelion.

Let’s denote R_0, as the distance between Sun And Earth at the moment when observers start measuring the movement, and r, the generic position of Earth. By taking a closer view at the very beginning of measurements onto this movement, for two dimensions, see **Fig. 3**. Say, when O’ and O coincide, a pulse of light is sent forming an angle β with X axis, see **Fig. 2**, and an angle γ between the tangential velocity v of O’ with Y axis as it is shown in **Fig. 3**. Let’s equally define θ**,** as the angle swept by radius r from $r = R_0$, to the new position r of the moving observer after a period of time dt. At this moment light pulse has reached point P. From **Fig. 2** and **3**, we can establish the following relationships:

$$dx' = k(dx - v.dt.\sin\gamma) \qquad dy' = k(dy - v.dt.\cos\gamma) \tag{8}$$

From the same graphs, we can establish that:

$$v.dt.\sin\gamma = d(R_0 - r.\cos\theta) \qquad v.dt.\cos\gamma = d(r.\sin\theta) \tag{9}$$

Given that the light speed is the same measured by any observer, it must fulfill:

$$dx^2 + dy^2 = c^2.dt^2 \qquad dx'^2 + dy'^2 = c^2.dt'^2 \tag{10}$$

Substituting dx', dy', by their expressions (8) and (9) into (10), similar expressions previously obtained for rectilinear movement are achieved:

$$dx' = \frac{dx - v.dt.\sin\gamma}{\sqrt{1-\frac{v^2}{c^2}}} \qquad dy' = \frac{dy - v.dt.\cos\gamma}{\sqrt{1-\frac{v^2}{c^2}}} \qquad dt' = dt.\frac{\sqrt{\left(\sin\gamma - \frac{v.u_x}{c^2}\right)^2 + \left(\cos\gamma - \frac{v.u_y}{c^2}\right)^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$u'_x = \frac{u_x - v.\sin\gamma}{\sqrt{\left(\sin\gamma - \frac{v..u_x}{c^2}\right)^2 + \left(\cos\gamma - \frac{v..u_y}{c^2}\right)^2}} \qquad u'_y = \frac{u_y - v.\cos\gamma}{\sqrt{\left(\sin\gamma - \frac{v..u_x}{c^2}\right)^2 + \left(\cos\gamma - \frac{v..u_y}{c^2}\right)^2}} \qquad (11$$

Namely, $$d\mathbf{r}'=\frac{d\mathbf{r}-v.d\mathbf{t}}{\sqrt{\mathbf{1}-\frac{v^2}{c^2}}};\qquad d\mathbf{t}'=\frac{d\mathbf{t}-\frac{v}{c^2}d\mathbf{r}}{\sqrt{\mathbf{1}-\frac{v^2}{c^2}}}\qquad \mathbf{u}'=\frac{d\mathbf{r}'}{dt'}\qquad (12)$$

These results show that the structure of differential VLT for curvilinear is the same previously viewed for rectilinear movement. Obviously, all this indicates that application of LLT to curvilinear movement is also valid, which will allow us to continue developing Relativity within only one theory.

The previous analysis was presented for the particular case at the beginning of measurements to differentiate the lengths of the differentials *dx* and *dx'*. The same situation can be displayed for the generic point P that the pulse of light is drawing in the space, where the same relationships are Also valid (see Fig. 4). Namely:

$$x'=\frac{x-\int v.dt.\sin\gamma}{\sqrt{1-\frac{v^2}{c^2}}}\quad\Rightarrow\quad dx'=\frac{dx-v.dt.\sin\gamma}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$y'=\frac{y-\int v.dt.\cos\gamma}{\sqrt{1-\frac{v^2}{c^2}}}\quad\Rightarrow\quad dy'=\frac{dy-v.dt.\cos\gamma}{\sqrt{1-\frac{v^2}{c^2}}}$$

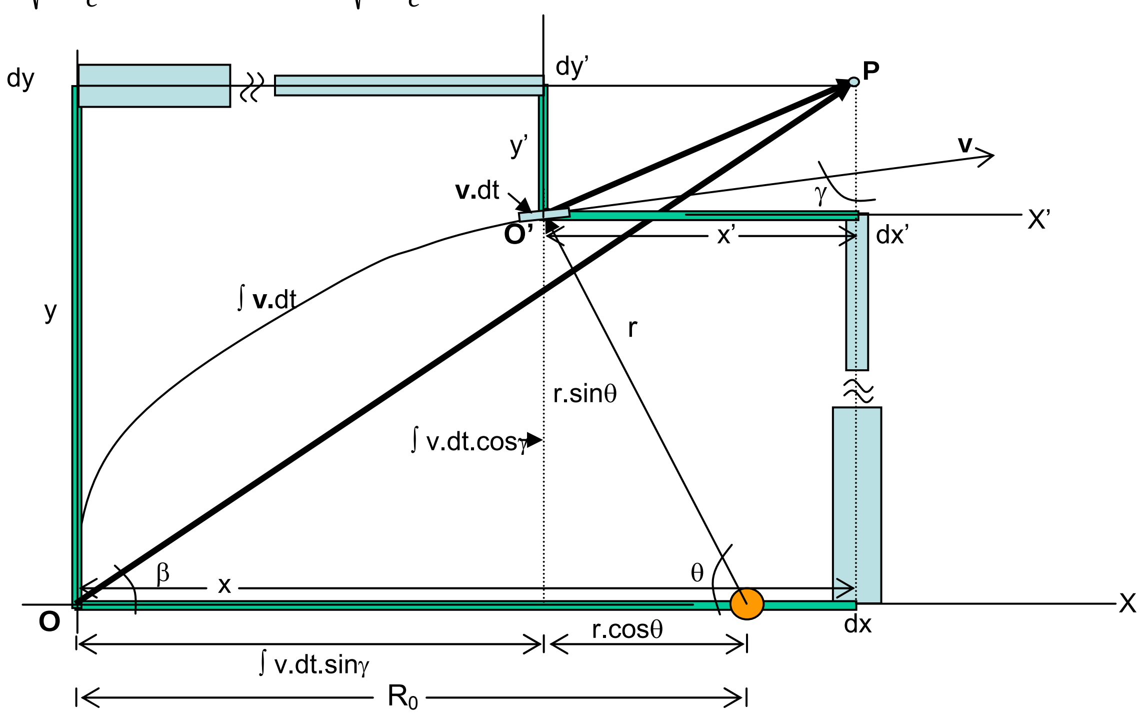

Fig. 4

Let's continue obtaining other dynamic transformations, for instance, that for angle between inertial systems. This magnitude emanate from the relation between curvilinear length of arc s and length of radius R. Because both magnitudes are lengths, Lorentz factors cancel out, and angle becomes invariant to LLT (this result is different to that of explained by Einstein in SRT [2]):

$$\alpha' = \frac{s'}{R'} = \frac{\dfrac{s}{\sqrt{1-\dfrac{v^2}{c^2}}}}{\dfrac{R}{\sqrt{1-\dfrac{v^2}{c^2}}}} = \frac{s}{R} \Rightarrow \alpha' = \alpha; \quad d\alpha' = \frac{ds'}{R'} = \frac{\dfrac{ds}{\sqrt{1-\dfrac{v^2}{c^2}}}}{\dfrac{R}{\sqrt{1-\dfrac{v^2}{c^2}}}} = \frac{ds}{R} \Rightarrow d\alpha' = d\alpha \tag{13}$$

In this way, angular velocity transforms as:

$$\omega' = \frac{d\alpha'}{dt'} = \frac{d\alpha}{dt.\sqrt{1-\dfrac{v^2}{c^2}}} = \frac{\dfrac{d\alpha}{dt}}{\sqrt{1-\dfrac{v^2}{c^2}}} \Rightarrow \omega' = \frac{\omega}{\sqrt{1-\dfrac{v^2}{c^2}}} \tag{14}$$

In next Review we will obtain the LLT of Force and other physical magnitudes.

V. CONCLUSION

We have observed that Local Lorentz Transformations (LLT) give us the true dynamical value of a physical magnitude whose rest value is known, namely LLT inform us about the real dependence a physical magnitude has on the speed of light and on its own speed in space. In this way, the Theory of Relativity stops being a mysterious and complex subject, understood only by a few individuals, to become something simple and reasonable, familiar to anyone, and revealing to us a new and simple physical law that governs the movement of the bodies in space.

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