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Galilean and Lorentz Transformations

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ABSTRACT: This paper discusses the derivation of Galilean and Lorentz Transformations (LT), in a way that allows revealing the weakness of the LT as we know them nowadays. We proceed to derive Galilean Transformations (GT) first for an one-dimensional configuration and later in a generalized manner, in order to follow the same presentation for LT to establish the differences with known LT. We have followed Franco's procedure to achieve this review.

KEYWORDS: Special Relativity, Galilean Transformations, Lorentz Transformations.

I. GALILEAN TRANSFORMATIONS

Galilean Transformations are obtained from a common sense analysis of the relative motion. For example, let's consider two observers, with all the equipment for doing measurements of length, time and velocities onto moving projectiles; the first observer located at the origin of coordinates of a fixed system O, and the second one located at the origin of coordinates of an inertial moving system O'. System O' moves at a constant velocity v, relative to O, such that X' and X axes are on the same line. The goal is to obtain relationships between the observer's measurements, such that they must be valid for any velocity of any projectile.

In order to arrive at the solution let's take as projectile a bullet.

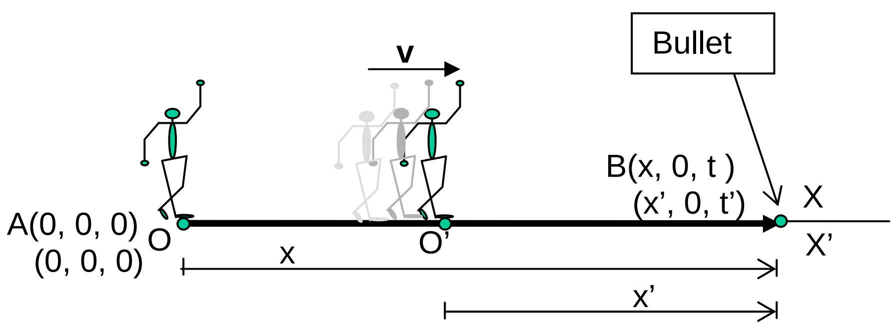

Fig. 1 **One-dimensional motion**

See the **Fig. 1** When, coming from $-\infty$, the origin O' of moving system coincides with fixed origin O, at $t = t' = 0$, and a bullet is shot parallel to X-axis, fixed observer at O measures a component x of the bullet displacement and the moving observer at O' measures a component x'. Given that components on axes Y and Z are null, common sense allow us establishing the following relationships, known as Galilean (one-dimensional) Transformations:

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$$x' = x - v.t; \quad y' = y = 0; \quad z' = z = 0; \quad t' = t.$$
$$v_{x'} = v_x - v \qquad (1)$$

As we see, the measurements of the speed of the bullet, are different for both observers. This is a logic result, in the following sense: For example, if the speed of the moving observer is the same as that of the bullet shot by the fixed observer, then moving observer will see the bullet as stationary in his space or his frame of reference (Suppose that gravity is not present). The same occurs when two vehicles are going parallel at the same speed; no one of the observers sees the other as stationary.

Let's represent this situation in a more general way. Referring now to the **Fig. 2** where each of the two observers, has all the equipment for doing measurements of length, time and velocities onto moving projectiles: one located at the origin of coordinates of a fixed system O, and the other located at the origin of coordinates of an inertial moving system O'. But now, system O' moves **along an inclined line** at a constant velocity v , relative to O, such that both observers are on the same line. The inclined line forms an angle β with the plane XY and its projection on this plane XY an angle α with the X-axis The axes of both systems maintain always parallel, say X parallel to X', Y parallel to Y' and Z parallel to Z'. Let $\mathbf{r}$ be the radio-vector of a rectilinear trajectory of the bullet, measured by the observer at O and Let $\mathbf{r}'$ be the radio-vector of the rectilinear trajectory of the bullet, measured by the observer at O'. The goal is to obtain relationships between observer's measurements. In order to arrive at similar relationships obtained for one-dimensional motion, taking as before a bullet as projectile, we will apply our known common sense, and in this way we will have the general expression of Galilean Transformations (spatial three-dimensional plus time) that should be valid for any projectile at any velocity:

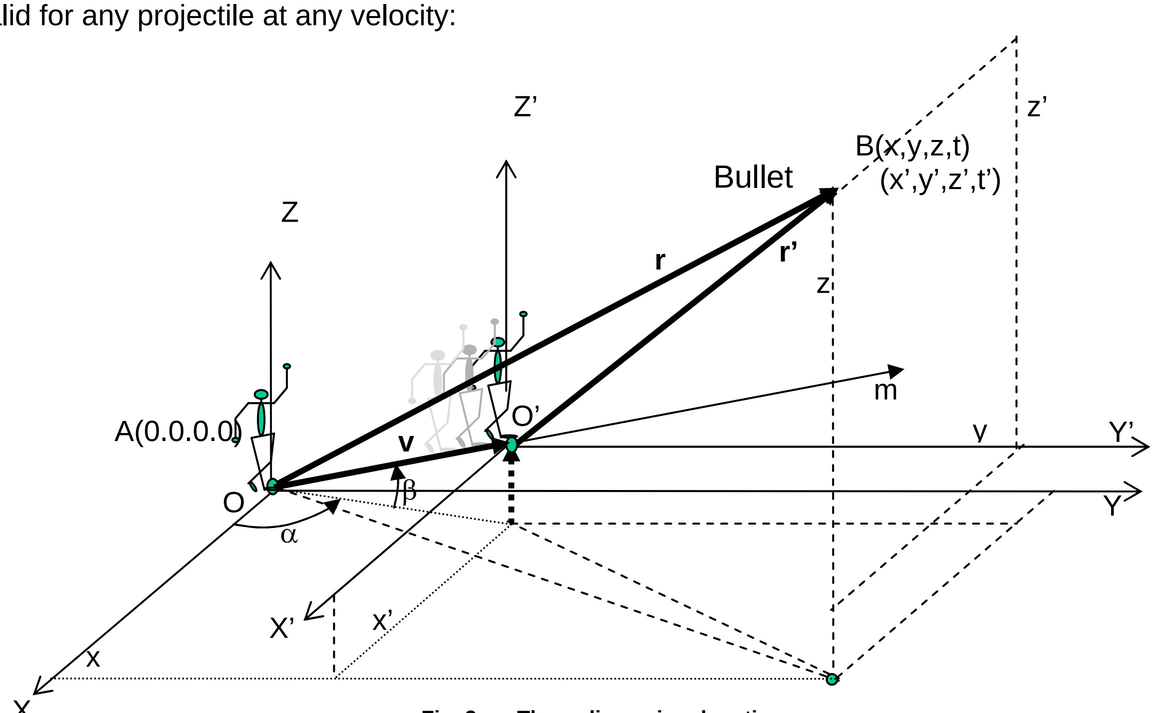

Fig. 2 **Three-dimensional motion**

By analyzing **Fig. 2** we can form the following relationships

$$\begin{aligned} x' &= x - v.t.\cos\alpha.\cos\beta \\ y' &= y - v.t.\sin\alpha.\cos\beta \\ z' &= z - v.t.\sin\beta \\ t' &= t \end{aligned} \qquad (2)$$

In this way we have arrived at the general Galilean Transformations by using our common sense and experience where we give for granted that time is the same for any observer and is independent of any other parameter. This indeed was the legacy of Aristotle, because he was who ordered all these physical and mathematical concepts 24 centuries ago, establishing and forming the scientific basement of our “common sense”.

Nevertheless, from the Maxwell’s four equations that synthesized all the electromagnetic knowledge in the nineteenth century, it could be concluded directly that the speed of light (an electromagnetic wave) is a universal constant, independent of the movement of the observer that measures it. This result brought about a big problem to the scientists of that century (1864), because the Galilean Transformations were not compatible with this constancy of the speed of light. As an example of these contradictions inside the very concept of light, in the sense that it exists if it has a speed c, and if not, it doesn’t exists; when Einstein was very young, under eighteen, he devised the following paradox of Galilean Transformations: if a person goes at the speed of light (see last equation of (1)), then he could observe at his side a *stationary* photon! Namely, a photon with null speed!. At that time, it was in those years 1995-1997, young Einstein thought, perhaps with knowledge of this discussion, that if the *stationary-photon* case is not possible then, on the contrary everyone must measure the same speed of light, as a universal constant!!.

On the other hand, unexpected results obtained in the famous experiment conducted by Albert Michelson and Edward Morley in 1887, according to which Earth did not move in any direction (!), under the interpretation induced at that time by Ether hypothesis, strongly motivated Dutch physicist Hendrick Antoon Lorentz and British physicist George Francis Fitzgerald, almost simultaneously during 1889-1890, to look for another way to interpret this result under the idea of a constant speed of light. They undertook this task by establishing relationships that preserved Maxwell equations to be the same in any inertial system (principle of relativity), and the speed of light with a constant value, measured by observers with different inertial movements. They established independently a set of transformations that implied the interdependency among time and space through the relative velocity of the moving system and the speed of light. After many controls and checks on these transformations as those of being consistent with Maxwell Equations, of preserving the constancy of the speed of light, of being valid not only for photons but for any projectile and also of being reduced to Galilean Transformations for low velocities respect to the speed of light, $v << c$, it led to discard the undoubted, untouchable and accepted, until that time, Galilean transformations, and to replace them by Lorentz Transformations (LT), so called, after they were displayed formally in 1904 by the very respected physicist H. A. Lorentz [1]. As it was previously said, it was known at that time that Galilean transformations although privileged as of “physical common sense”, neither preserved the constancy of light speed nor were consistent with Maxwell Equations. After Albert Einstein’s solid arguments destroyed Ether hypothesis in his seminal work about Relativity in 1905 [2], LT became central for the Special Theory of Relativity (STR) [3]. As we will see in next sections, there exists a problem: LT were developed only for a one-dimensional motion, but later, under some false assumptions (it will be demonstrated in next sections), were generalized to any number of dimensions (!). By the way, it was also A. Michelson one of the first light scientists that most exactly measured the speed of light (299.853±60 km/s in 1883, and 299.796±4 km/s in 1925), after many others that tried the same goal. Currently the used value of the speed of light is 299 792 458 m/s. The constancy of the speed of light was defined as a constant, not to be measured again, in the 17th Conférence Générale des Poids et Mesures, in 1983, and used for defining the meter: "The metre is the length of the path traveled by light in vacuum during a time-interval of 1/299 792 458 of a second".

II. LORENTZ TRANSFORMATIONS

A simple way modernly used to arrive at the known LT (one-dimensional), under the postulate that velocity of light is constant and independent of the source in any inertial frame and that the laws of physics are the same in all inertial frames is, as before, to consider two observers, with all the equipment for doing measurements of length, time and velocities onto moving projectiles; the first one located at the origin of coordinates of a fixed system O, and the second one located at the origin of coordinates of an inertial moving system O'.

In order to respect the previous restrictions, let each observer not know the presence of the other. Then, each one will see him as static or fixed and the other moving.

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System O' moves at a constant velocity v, relative to O, such that X' and X axes are on the same line, see **Fig. 3**. The goal is to obtain relationships between the observer's measurements, such that they must be valid for any velocity of any projectile including that of photons, c. In order to arrive at a solution taking into account such conditions, we will take as projectile a pulse of light. As a control, resulting transformations should also be consistent with Galilean transformations for inertial systems with relative velocity $v << c$.

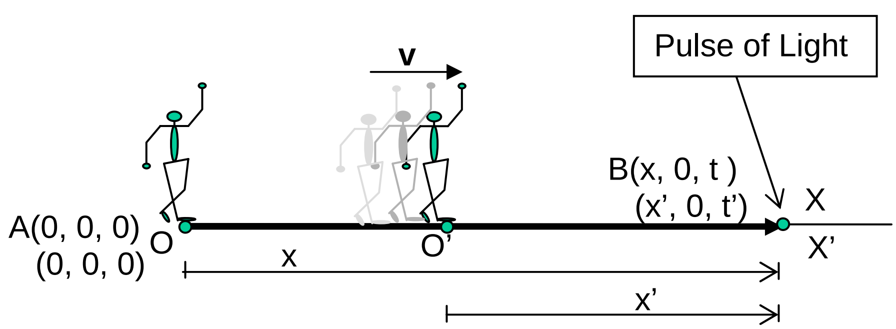

Fig. 3 One-dimensional motion

By doing a Galilean reasoning, but using a scaling factor k, to be calculated, for mathematically preserving the constancy of the speed of light, c, it is established the following relationship for the X-axis component of the light pulse:

$$x' = k.(x - v.t) \tag{3}$$

Under these conditions, the light pulse also must fulfill the following relationships for any observer:

$$x' = c.t' \quad \Rightarrow \quad t' = \frac{x'}{c} \qquad\qquad x = c.t \quad \Rightarrow \quad t = \frac{x}{c} \tag{4}$$

Or, under the same conditions, for any projectile traveling at velocity $u_x \leq c$:

$$x' = u'_x.t' \quad \Rightarrow \quad t' = \frac{x'}{u'_x}; \qquad x = u_x.t \quad \Rightarrow \quad t = \frac{x}{u_x}; \qquad u_x \neq u'_x \tag{5}$$

Substituting relations from equations (4) into equation (3), LT for time is readily obtained:

$$c.t' = k.\left(c.t - v.\frac{x}{c}\right) \qquad \Rightarrow \qquad t' = k.\left(t - \frac{v}{c^2}.x\right) \tag{6}$$

Now, change the observer's role, namely, start considering O' fixed and O, the moving system (remember that each one considers himself as fixed). Under this configuration it is clear that the observer at O' will see the system O moving at velocity $-v$, namely, going in opposite sense to the light pulse. Reasoning as in the original situation, a pulse of light is sent when O and O' coincide, and a similar relation to that found in (1) is constructed through the same constant k **(**see **Fig. 4)**:

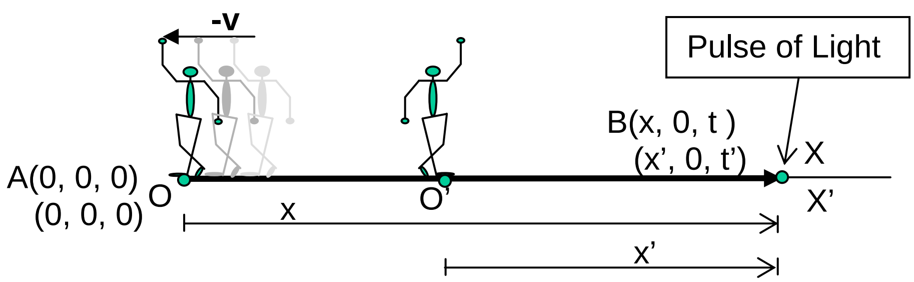

Fig. 4 One-dimensional motion

$$x = k.(x' + v.t') \tag{7}$$

Because these equations must be valid in any situation, from equations (3), (4) and (7), the following relationships hold:

$$x' = k.(x - v.t) = k.\left(x - \frac{v.x}{c}\right) = k.x.\left(1 - \frac{v}{c}\right)$$
$$x = k.(x' + v.t') = k.\left(x' + \frac{v.x'}{c}\right) = k.x'.\left(1 + \frac{v}{c}\right)$$

Multiplying both equations, the value of factor k is obtained:

$$x'.x = k^2.x.x'.\left(1-\frac{v^2}{c^2}\right) \qquad \Rightarrow \qquad k=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{8}$$

As a check: For $v << c \;\Rightarrow\; k \cong 1;\quad x' \cong x - v.t;\; y' = y = 0;\; z' = z = 0;\; t' \cong t$. Namely, as it was expected, under these conditions LT are reduced to those of Galileo.

In this way the set of one-dimensional transformation equations between both systems of coordinates were obtained. The next step usually found in the literature consists in extending the validity of these relationships to any movement of O', but maintaining the moving system on the X-axis as is viewed in **Fig. 5**, using *arguments* (assumptions) as these: "by the geometry of the problem" or" by the Isotropy postulate", the following "general" relationships are obtained:

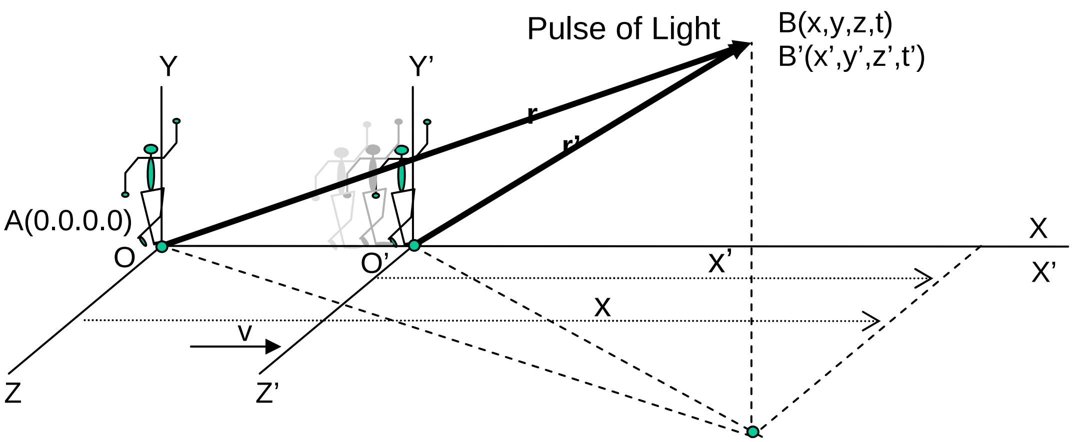

Fig. 5 "Three-dimensional motion"

$$x' = \frac{x - v.t}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \boxed{y' = y \qquad z' = z} \qquad t' = \frac{t - \frac{v.x}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = t.\frac{1-\frac{v}{c^2}.u_x}{\sqrt{1-\frac{v^2}{c^2}}} \tag{9}$$

Dividing by time, velocity transformations u of light pulse are,

$$u'_x = \frac{u_x - v}{1-\frac{v.u_x}{c^2}} \qquad \boxed{u'_y = \frac{u_y.\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v.u_x}{c^2}} \qquad u'_z = \frac{u_z.\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v.u_x}{c^2}}} \tag{10}$$

These are the famous and well known Lorentz Transformations, currently used.

First of all, it is worth mentioning that the second and third expressions in (9) do not come from equations or relationships but from *assumptions*. Again, the second and third expressions in (10) are direct consequence of such assumptions. It is important to emphasize that LT in (9) and (10) are currently accepted all over the world since the Special Theory of Relativity arrived at a century ago.

One of the aspects that disenchant to those trying to see more inside LT is that it is impossible to find their derivation in a generalized form for any type of movement of the inertial system O'. In fact, as far as we know, in all the publications taking up this subject, the moving system O' is always confined to move on the X axis, extrapolating by "common sense" or any other postulate, to the general movement of O' [1], [2], [3], [4], [5], [6], [7]; ¿Why? ¿Why cannot the system O' be generally presented as moving along, for example, an inclined trajectory, not coinciding with any axis, in order to describe the situation for any point including those on the axes? May be it can be understood that such configuration introduces problems for establishing the previously mentioned assumptions, -until now untouchables and accepted-? In next review published in this journal, Generalized Vectorial Lorentz Transformations, it is demonstrated that assumptions $y' = y$ and $z' = z$ are unnecessary and hence, groundless and wrong.

Obviously, Lorentz objective was to correct Galilean transformations in order to obtain general transformations that preserve the constancy of the light speed and also to be consistent with Maxwell equations so that they remained the same in any inertial system, but, as far as I know no one has corrected the mentioned assumptions. In the next paper, LT are generalized by taking the moving system O' to move on an inclined line, the same as we did before within the derivation of Galilean Transformations. This configuration allowed us to discover new features about time and took us to develop LT in a simple and consistent way, without doing any assumptions.

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