ABSTRACT: In previous work it was shown that assumptions, \( y' = y \) and \( z' = z \), within Lorentz Transformations were needless, and therefore groundless. Because of such assumptions, Lorentz Transformations (LT) depend on the body’s spatial orientation, i.e. the well-known transverse and longitudinal transformations of magnitudes, characterized by different scaling factors. On the contrary, the development of LT without assumptions, brought about new transformations that do not depend on spatial orientation, and a unique mass definition was devised, \( m = \frac{M_0}{\sqrt{1 - \frac{v^2}{c^2}}} \). As it is known, Einstein arrived at two definitions: transverse mass \( m_T = \frac{M_0}{\sqrt{1 - \frac{v^2}{c^2}}} \) and longitudinal mass \( m_L = \frac{M_0}{1 - \frac{v^2}{c^2}} \). This latter coincides with our obtained mass definition. In the current work, based on the unique definition of mass, new expressions of Energy and Momentum were derived. As an interesting result, it was encountered that Einstein’s equation \( E = m_c^2 \) is only valid for particles with null mass at rest (i.e. photons). In contrast, it was noticed that \( E = m_c^2 \) works as a very good approximation in energy calculations for bodies with non-null mass at rest, at speeds less than two thirds that of light. By applying the new expression of Energy, a modified Schrödinger Equation was obtained.


I. INTRODUCTION

The concept of variation of a mass \( m \) with its velocity \( v \), through its rest mass \( M_0 \) and the universal constant speed of light \( c \), in rectilinear motion, given by \( m = \frac{M_0}{\sqrt{1 - \frac{v^2}{c^2}}} \), was originally established by Einstein in the section §10, of his seminal paper about relativity on June 30th, 1905 [1]. Doubtlessly, the mass dependence of a body on its velocity was one of the major outcomes of the Special Theory of Relativity. Additionally, in the same section §10, Einstein indirectly sets up (with another notation) his referred famous equation, through the derivation of the kinetic energy of the electron, \( K = m_c^2 - M_0c^2 \). In where \( K \) denoted the Kinetic energy of the electron and \( m = \gamma M_0 \) its relativistic mass, for \( \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \), moving at velocity \( v \). Later, on September 27th 1905, in a second paper he formally presented this result by establishing: “the mass of a body is a measure of its
energy contents”, referring this comment to the previous definition of energy [2]. However, “The first record of the relationship of mass and energy explicitly in the form $E = mc^2$ was written by Einstein in a review of relativity in 1907”, After that, “Relativistic mass came into common usage in the relativity text books of the early 1920s written by Pauli, Eddington and Born”[3].

Some light criticisms about the Einstein’s first derivation of relativistic mass in 1905 can be read in [3] [4] [5] [6] [7].

It is important to observe that Einstein did not derive his energy equation from the transverse mass definition as it is presented in the First Part of this work. In its place, he originally started from the assumption of equating the energy withdrawn from the electrostatic field, to the “energy of motion” of the electron. The inter-conversion mass-energy coined by Einstein in his equation, $E = mc^2$, has been accepted and interpreted in different ways by scientists of any kind, practically since that year 1905 when it was formally presented to the scientific world in the German publication Annalen der Physic. On the other hand, there is an interesting work of Mendel Sachs in 1973 that calls the attention on the non-consideration of the change of nuclear configuration energies within the atom in a re-examination of such equation [8].

In the present work it is demonstrated that $E = mc^2$ is valid only for photons and therefore it should not be always applicable for particles with non-null rest mass.

This work is presented in the following order: In Section II, is shown a direct way for obtaining the Einstein's equation of energy, $E = mc^2$, starting from the Einstein's transverse mass definition (incorrect, according to the development done in [9]). In Section III, the new definition of mass given in [9] was used instead and a new Energy Equation was obtained. In Section IV a modified Schrödinger Equation is found based upon the new definition of relativistic Energy.

### II. ENERGY DERIVATION FROM EINSTEIN’S MASS.

In the next paragraphs we will present a simple way of deriving the Einstein’s energy definition from the transverse mass definition, following a similar procedure to that appeared in [10].

The classical Newton’s second law, in its modern presentation, correctly establishes that the net Force $\mathbf{F}$ exerted on a mass $m$, equals the derivative relative to time of its Linear Momentum, $\mathbf{p}$ (the product of its mass $m$ times its velocity $\mathbf{v}$), $\mathbf{F} = \frac{d(m \mathbf{v})}{dt}$. By considering the mass as a variable, in this development Newton's 2nd Law becomes:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d(m \mathbf{v})}{dt} = m \frac{d\mathbf{v}}{dt} + \mathbf{v} \cdot \frac{d(m)}{dt}$$

(1)

The differential of Kinetic Energy $dK$ is defined as the work done by a force $\mathbf{F}$ in order to make a mass $m$ from rest have a displacement $d\mathbf{s}$. Applying the vectorial identity,

$$d(\mathbf{A} \cdot \mathbf{A}) = 2 \mathbf{A} \cdot d\mathbf{A} = d(A^2) = 2 A \cdot dA \Rightarrow \mathbf{A} \cdot d\mathbf{A} = A \cdot dA,$$

it follows:
Taking derivatives of Einstein’s transverse mass definition, we will obtain:

\[ m = \frac{M_0}{\sqrt{1 - \frac{v^2}{c^2}}} \implies \frac{dm}{M_0} = \frac{c^2.dm}{(c^2 - v^2)^{\frac{3}{2}}} = \frac{1}{\left(\frac{c^2}{c^2 - v^2}\right)^{\frac{1}{2}}} \frac{c^2.dm}{(c^2 - v^2)^{\frac{1}{2}}} \]

Operating, using (2) and simplifying:

\[ \frac{dm}{m} = \frac{c^2.dm}{(c^2 - v^2)^{\frac{3}{2}}} \implies c^2.dm = v^2.dm + m.v.dv \implies c^2.dm = dK \]

From equations (2) and (4), and integrating from rest mass, \( M_0 \), to a new generic state of mass \( m \), Einstein’s Kinetic energy expression is readily obtained:

\[ K = \int (m.v.dv + v^2.dm) = \int_{M_0}^{m} c^2.dm \implies K = m.c^2 - M_0.c^2 \]

As a control of this, the expression (5) should reduce to the well-known Newton’s kinetics energy expression: \( K = \frac{1}{2}.m.v^2 \), for \( v << c \). Einstein showed this fact by expanding equation (5), i.e.,

\[ K = \frac{M_0.c^2}{1 - \frac{v^2}{c^2}}, \text{ as a binomial series of the type:} \]

\[ (1 + x)^{\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{1.3}{2.4}x^2 - \frac{1.3.5}{2.4.6}x^3 + ... \quad -1 < x \leq 1 \quad \text{for} \quad x = \frac{v^2}{c^2} \]

\[ \left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}} = 1 + \frac{1}{2}\left(\frac{v^2}{c^2}\right) + \frac{1.3}{2.4}\left(\frac{v^2}{c^2}\right)^2 + \frac{1.3.5}{2.4.6}\left(\frac{v^2}{c^2}\right)^3 + ... \quad \frac{v^2}{c^2} \leq 1 \]

Substituting and discarding the terms divided by \( c^2 \), raised to any exponent:

\[ K = M_0 \left[ \left( c^2 + \frac{1}{2}.v^2 + ... \right) - c^2 \right] \approx \frac{1}{2}.m.v^2 \quad \text{for} \quad M_0 \approx m \]

Remembering that Total Energy for a free particle, \( E \), is the Kinetic energy \( K \) plus the internal energy \( E_0 = M_0.c^2 \), and by operating on equation (5) it follows:

\[ E = (m - M_0)c^2 + M_0.c^2 \implies E = m.c^2 \]

Thus, certainly Einstein’s Energy equation in (7) is a direct consequence of the Einstein’s transverse mass definition, and although “transverse energy” does not sound quite well, everything has remained until now “OK” with this definition. The “only” problem is that such mass definition, recently was determined as not correct in [9]. The simplicity of the previous derivation makes it beautiful. In author’s opinion, Einstein was a victim of this illusion. It is worth to insist that Einstein’s procedure
used to arrive at his famous equation was based on the assumption of which the energy withdrawn from the electrostatic field develops into the energy of motion of an electron slowly accelerated. This is something completely different to the formal derivation used here (also incorrect because it was done based upon an erroneous concept of mass). In the next part we will start using the correct mass definition obtained in reference [9], for deriving new and exact definitions of Energy and Momentum.

### III. NEW ENERGY DEFINITION.

From our previous work, the correct and unique definition of mass was obtained by working under the Local Lorentz Transformations (LLT) and by the application of the Angular Momentum Conservation Law to an inertial curvilinear movement of a mass attracted by another one [9]. Such mass definition was derived and established without any assumption. As it was referred before, this expression was:

\[
m = \frac{M_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \tag{8}
\]

It is noteworthy that Einstein also obtained the same relationship, under a different procedure and called it "longitudinal mass" [1] to distinguish it from well known (transverse) mass definition. Later, this former concept little by little was left out, until apparently it was discarded from his work.... (!).

Relativistic Mass, as a unique definition, expressed in equation (8) came up as product of a simple and rigorous derivation, without assumptions, and therefore it should not be considered in any case as a proposal, assumption or suggested definition, namely, it is a result!

It can be observed also that the new relativistic mass definition ratifies the dependence of the body’s mass on its velocity, as it is reflected by equation (8). In the next section a different definition of Energy directly derived from the referred new mass definition will be obtained.

#### A. New Definition of Kinetic Energy

Let’s consider a mass that moves describing a curvilinear path at velocity \( \mathbf{v} \). As it was previously indicated, kinetic energy expression is given by:

\[
dK = \mathbf{F} \cdot d\mathbf{r} = \frac{d(m, \mathbf{v})}{dt} \cdot d\mathbf{r} = d(m, \mathbf{v}).\frac{d\mathbf{r}}{dt} = d(m, \mathbf{v}).\mathbf{v} \tag{9}
\]

\[
dK = m.d\mathbf{v} \cdot \mathbf{v} + \mathbf{v} \cdot d\mathbf{v}.m = m.d\mathbf{v} \cdot \mathbf{v} + v^2.dm
\]

Observe that in the new mass definition in (8), \( M_0 \), the rest mass, is a constant magnitude and the only variable is its speed, \( \mathbf{v} \). Accordingly, the differential of mass can be derived as follows:

\[
m = \frac{M_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} = \frac{M_0.c^3}{\left(c^2 - v^2\right)^{\frac{3}{2}}} \tag{10}
\]
\[ dm = 3. M_0 c^3 \cdot \frac{v.dv}{(c^2 - v^2)^{\frac{5}{2}}} = 3. \frac{M_0 c^3}{(c^2 - v^2)^{\frac{5}{2}}} \cdot \frac{v.dv}{(c^2 - v^2)^{\frac{5}{2}}} = 3. m_0 c^3 \cdot \frac{v.dv}{(c^2 - v^2)^{\frac{5}{2}}} \]

\[ dm = 3. m_0 c^3 \cdot \frac{v.dv}{(c^2 - v^2)^{\frac{5}{2}}} \]

A similar procedure to that used in previous section is repeated here for obtaining the new expression of energy. Operating on the first equation in (11) and trying to recreate the last energy equation in (9), it follows that:

\[ \frac{c^2 dm}{3} - \frac{v^2 dm}{3} = m_0 v dv \quad \Rightarrow \quad \frac{c^2 dm}{3} = m_0 v dv + \frac{v^2 dm}{3} \]

By adding, \( \frac{2v^2 dm}{3} \), to both members of the last relationship, we can construct the known kinetic energy expression, articulated in equation (9):

\[ \frac{c^2 dm}{3} + \frac{2v^2 dm}{3} = m_0 v dv + \frac{v^2 dm}{3} + \frac{2v^2 dm}{3} = m_0 v dv + v^2 dm = dK \]

\[ dK = \frac{c^2 dm}{3} + \frac{2v^2 dm}{3} \]

As we expect, expression (12) equals the differential of kinetic energy. Thus, substituting in the second term of the right member of equation (12) the second expression of \( dm \) that appears in (11), it follows:

\[ dK = \frac{c^2 dm}{3} + \frac{2v^2 dm}{3} m_0 c^3 \cdot \frac{v.dv}{(c^2 - v^2)^{\frac{5}{2}}} \]

In this way we have in equation (13) the first term at right depending only on the mass \( m_0 \), and the second one depending only on the velocity \( v \).

Before continuing, for the sake of simplicity, let’s now refer for a moment to the Earth movement around the Sun, for detecting the different presentations mass can have (this can be thought referred to any other general case of two attracting masses, one of them considered fixed and respect to which all measurements are done). Let’s establish that the point for starting measurements will be Perihelion, the closest point between Earth and Sun. At that moment, Earth has a velocity \( V_0 \), and an “initial” mass denoted by \( m_0 \), which is different from the rest mass, \( M_0 \), because \( V_0 \neq 0 \). In fact, \( m_0 = \frac{M_0}{1 - \frac{V_0^2}{c^2}} \). When Earth occupies any other position on its elliptical path, it will have another generic velocity \( v \), and of course another generic mass value, denoted as:
After this parenthesis, where the mass can be recognized (in general) in three distinct manners, let’s do the integration of the differential of kinetic energy indicated in equation (13), between \((V, m)\) and \((v, m)\). Thus, after some simplifications, the general (rectilinear and curvilinear) equation for a change in Kinetic Energy follows:

\[
K - K_0 = m(2v^2 - c^2) - m_0(2V^2 - c^2)
\]  

(14)

See that \(K_0\) is that kinetic energy at \(V\), when Earth is at perihelion.

By operating upon equation (14), in order to have a suitable expression for later expanding their elements in binomial series:

\[
K - K_0 = m.v^2 + m(V^2 - c^2) - m_0.V_0^2 - m_0(V_0^2 - c^2)
\]

\[
K - K_0 = m.v^2 - m_0.V_0^2 - m.c^2\left(1 - \frac{v^2}{c^2}\right) + m_0.c^2\left(1 - \frac{V_0^2}{c^2}\right)
\]

Reordering and Substituting by mass expressions:

\[
K - K_0 = \frac{M_0.v^2}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} - \frac{M_0.V_0^2}{\left(1 - \frac{V_0^2}{c^2}\right)^{\frac{3}{2}}} - \frac{M_0.c^2}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}} + \frac{M_0.c^2}{\left(1 - \frac{V_0^2}{c^2}\right)^{\frac{1}{2}}}
\]

Expanding in binomial series, as before, by using:

\[
(1 + x)^{\frac{1}{2}} = 1 - \frac{1}{2}.x + \frac{1.3}{2.4}.x^2 - \frac{1.3.5}{2.4.6}.x^3 + .... - 1 < x \leq 1
\]

\[
(1 + x)^{\frac{3}{2}} = 1 - \frac{3}{2}.x + \frac{3.5}{2.4}.x^2 - \frac{3.5.7}{2.4.6}.x^3 + .... - 1 < x \leq 1
\]

Putting \(v << c\) for reducing to the Newtonian case where \(M_0 \equiv m\), and discarding those terms divided by \(c^2\), raised to any exponent, it is obtained:

\[
K - K_0 = M_0\left[v^2 + ... - \left(V_0^2 + ...\right) - \left(c^2 + \frac{1}{2}.v^2 + ...\right) + \left(c^2 + \frac{1}{2}.V_0^2 + ...ight)\right]
\]

The change in kinetic energy reduces consistently to the known Newtonian expression:
\[ K - K_0 \approx \frac{1}{2}m\nu^2 - \frac{1}{2}m\nu_0^2 \]  

(15)

The Kinetic Energy of a body in any curvilinear movement \( K \), when starting from rest: \( \nu_0 = 0 \), and, \( m_0 = M_0 \), until it gets a velocity \( \nu \), and mass \( m \), equation (14) reduces consistently to:

\[ K = m(2\nu^2 - c^2) + M_0c^2 \]  

(16)

Operating on this equation, in order to check its consistency:

\[ K = m\nu^2 + m(\nu^2 - c^2) + M_0c^2 = m\nu^2 - m\frac{c^2(1 - \frac{\nu^2}{c^2})}{2} + M_0c^2 \]

\[ K = \frac{M_0\nu^2}{3} - \frac{M_0c^2}{(1 - \frac{\nu^2}{c^2})^2} + M_0c^2 = M_0\left[(\nu^2 + ...)-\left(c^2 + \frac{1}{2}\nu^2 + ...ight)c^2\right] \]

This equation also reduces, for \( \nu << c \), and \( M_0 \approx m \), to the Newtonian expression of kinetic energy of a body starting from rest:

\[ K \approx \frac{1}{2}m\nu^2 \]  

(17)

But, nature is very tricky because as we have seen before Einstein’s expression of total energy, \( E - E_0 = m.c^2 - m_0.c^2 \), also meets all these simplifications! These blurred results led to the general acceptance of the transverse mass definition and of the Einstein’s equation \( E = mc^2 \). The crucial point in this confusion was finally solved by the obtaining of the correct and unique definition of mass in [9].

As it can be observed, the new definition (14) applies for any body in any situation, with or without rest mass, and takes into account, noticeably and explicitly, all the velocities involved.

**B. Total Energy**

Let’s continue with our task of construction all the new definitions of energy. Remembering that Total Energy, for a free particle, is the summation of Kinetic Energy starting from rest (in its simpler presentation), previously obtained, plus Internal Energy, it follows:

\[ E = K + M_0c^2 = m(2\nu^2 - c^2) + M_0c^2 + M_0c^2 = 2M_0c^2 - m(c^2 - 2\nu^2) \]  

(18)

Let’s check. By doing, \( \nu = 0 \Rightarrow m = M_0 \), then Total Energy is reduced consistently to the internal energy, \( E = M_0c^2 \). Thus, the equation (18) is a general expression for any body with moving mass \( m \), starting from rest.
As it is well-known, photons do not have rest mass. Also, they do not exist for any other speed different from \( c \); but when they exist, each one of them does have a mass \( m \) and a constant velocity equal to \( c \). So, applying these conditions into equation (18) it is obtained that the energy of a Photon is \( E = m_c^2 \). So, from this development, is concluded that the equation \( E = m_c^2 \), is valid only for photons or particles with null rest mass. It is not valid for bodies whose rest masses are different of zero. So, the general Total Energy expression for any body, moving and starting from \( V_0 = 0 \), is that of the equation (18), and the particular version for photons is \( E = m_c^2 \).

A comparison between \( E = 2M_0c^2 - \frac{M_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \) and \( E = \frac{M_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \), the new Energy definition and Einstein’s Total Energy in function of transverse mass, considered as if it were valid for particles with non-null rest mass, respectively, is displayed below:

![Graph comparing new definition of Energy with Einstein's Energy definition.](image)

As it can be observed, both curves are very close until the body is moving at approximately a velocity of 200,000 KM/sec. From this graph, in our opinion it will be difficult to design an experiment to detect experimentally the coincidence of the correct values with Einstein’s or our’s curves. May be, if it were possible, the experiment should be conducted instead to detect the difference between the Einstein’s mass definition, \( m = \frac{M_0}{\sqrt{1 - \frac{v^2}{c^2}}} \), and that obtained in [9], \( m = \frac{M_0}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}} \), in where the difference is little bit more significant at smaller speeds than the previous ones. As an illustration a comparative graph is given next:
C. Linear Momentum

Likewise as Energy was redefined, we will do the same with Linear Momentum. By using the new mass definition, Momentum will be then:

\[
p = m.v = \frac{M_0.v}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}}\]

By substituting rest mass, \(M_0 = m\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}\), in equation (18), Energy and mass are related as follows:  

\[
E = 2.M_0.c^2 - m.(c^2 - 2.v^2) = 2.m.c^2\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}} - m.(c^2 - 2.v^2) 
\]

Introducing mass as function of Energy in (18), we have another expression for linear momentum:

\[
p = m.v = \frac{E}{2.c^2\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}} - (c^2 - 2.v^2)} \cdot v \tag{20}
\]
For instance, Momentum for the case of Photon, where, \( v = c \) and \( M_0 = 0 \), is reduced to the known relationship:

\[
p = \frac{E}{c}
\]  

(21)

An expression of the Energy-Momentum can also be encountered, and it is:

\[
E = 2.M_0.c^2 + p.v - c.\sqrt{M_0^2.c^2 + p^2\left(1 - \frac{v^2}{c^2}\right)}
\]  

(22)

Let’s check. By substituting the Momentum expression (19) in energy equation (22) the more at right following expression of energy is obtained:

\[
E = 2.M_0.c^2 + m.v^2 - m.c^2\left(1 - \frac{v^2}{c^2}\right) = 2.M_0.c^2 + p.v - \frac{M_0.c^2}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}}
\]

As it has been shown, the new definitions for Mass, Energy and Momentum are consistent with known expressions (for photons) of “modern” and Newtonian physics.

**IV. QUANTUM MECHANICS AND THE NEW RELATIVISTIC ENERGY DEFINITION.**

As it should be expected, this new definition of energy entails interesting consequences for quantum mechanics. In author’s opinion, the reason to use the classical equation of energy in quantum theory, in addition to give good experimental results, is that Einstein’s relativistic expression of kinetic energy does not depend directly and explicitly on the velocity of the particle as it can be encountered in Newtonian mechanics (which allows the direct relationship between the classic Hamiltonian with the Quantum operator): It introduces mathematical complications, difficult to handle by quantum mechanic physicists, although these problems were solved by P. A. M. Dirac [11].

For example, because in our study, the kinetic energy of a particle starting its movement from rest, depends directly and explicitly on the velocity of the particle, we can write:

\[
K = 2.m.v^2 - m.c^2 + M_0.c^2 = 2.\frac{p^2}{m} - c^2(m-M_0)
\]  

(23)

Forming total energy \( E \), by including the kinetic, the internal and the potential energy, for ensuring that \( E \) preserves constant in any case (including those cases where part of the internal energy converts to any other kind of energy, or there are energy-mass interchanges), and acquiring the expression of momentum, we have:

\[
E = K + M_0.c^2 + E_p = 2.m.v^2 - m.c^2 + 2.M_0.c^2 = 2.\frac{p^2}{m} - c^2(m-2.M_0) + E_p
\]  

(24)
This result recalls us the “classical” Hamiltonian. By using our relativistic expression in (24), for the case of three dimensions in the same way as we used the Classical Hamiltonian, we can re-define the Relativistic Hamiltonian as:

\[ H_{\text{relativistic}} = 2 \frac{P^2}{m} - c^2 (m - 2M_0) + E_p(r) \]  

(25)

From equation (24) we can obtain a suitable expression of the linear momentum:

\[ p^2 = \frac{E - E_p + c^2 (m - 2M_0)}{2} m \]  

(26)

Let’s do the following exercise: with the relationship (26) we will try to obtain our version of the Schrödinger Equation. For such task we are going to use the analogy between the Wave and Schrödinger Equations. I know that this way of introducing Schrödinger Equation is not so rigorous, but, in author’s opinion, this was used by Schrödinger in 1928 to obtain his equation, trying to correct and to extend the wave-particle interpretation put forward in 1926 by De Broglie. In this way, author insists that it is very illustrative and simple to understand. By the way, it is noteworthy that Schrödinger was opposed at that time, 1930, to the quantum interpretation given by Bohr [12]. Anyway, the intention here is to obtain direct results through simple analogies with quantum mechanics for the sake of synthesizing the applicability of deduced expressions, but at no moment the author wants to coin that this is a meticulous form to derive the equation of Schrödinger. This showing is only for quickly promoting the obtaining of new relativistic operators in Quantum Mechanics, given the direct and explicit dependency on velocity of our energy expression.

Let \( \gamma \) be the frequency of the wave, \( \lambda \) be its wavelength, \( k \) be the wave number, where \( k = \frac{2 \pi}{\lambda} \) and be \( h \) the Planck constant, and \( \hbar = \frac{h}{2 \pi} \).

By associating a one-dimensional periodical wave, given by \( \xi = \xi_0 \sin k(x - vt) \) to a particle that travels at a velocity \( v \) along the X-axis, the wave equation for this case becomes:

\[ \frac{d^2 \xi}{dx^2} + k^2 \xi = \frac{d^2 \xi}{dt^2} + \frac{p^2}{\hbar^2} \xi = 0, \]  

(27)

The velocity of the particle \( v \), which is the same of its associated wave with wavelength \( \lambda \) and frequency \( \gamma \), holds \( v = \gamma \lambda \). Let’s take the assumption that the product of the wavelength \( \lambda \) times the linear momentum \( p \) equals the Planck constant: \( h = p \lambda \) \( \Rightarrow \) \( p = \hbar \frac{1}{\lambda} = h \frac{2 \pi}{\lambda} = h k \), as it was established indirectly by Planck, in order to obtain the relationship between momentum \( p \) and the wave number \( k \). On the contrary, in this analysis of waves associated to particles with non-null mass at rest we have checked with our energy equations and concluded that the energy of a moving particle can not be considered equal to the Planck constant times its wavelength, \( E \neq h \gamma \), as it is for...
photons, without getting to fall in contradictions. Thus, by introducing the new momentum expression of equation (26) into equation (27), we come up with the Schrödinger spatial equation in one dimension:

\[- \frac{2\hbar^2}{m} \frac{\partial^2 \xi}{\partial x^2} + \left[ E_p - c^2 \left( m - 2M_0 \right) \right] \xi = E \xi \]  

(28)

To arrive at the general expression of Schrödinger equation, we are following a similar procedure to that presented by Alonso & Finn in [13]. Thus, we will be concerned in looking for a wave function that depends on the space and the time, which after deriving it with respect to the time and space we would have expected to obtain the spatial equation (28). The General Equation encountered by Schrödinger was:

\[- \frac{2\hbar^2}{m} \frac{\partial^2 \psi}{\partial x^2} + \left[ E_p - c^2 \left( m - 2M_0 \right) \right] \psi = j \hbar \frac{\partial \psi}{\partial t} \]  

(29)

A function that fulfills those requirements is a product of two separate variables:

\[ \psi(w, t) = \xi(x)e^{-\frac{i E_p t}{\hbar}} ; \text{ where, } \frac{\partial \psi}{\partial t} = -\frac{j}{\hbar}E \xi(x)e^{-\frac{i E_p t}{\hbar}} ; \text{ and } \frac{\partial^2 \psi}{\partial x^2} = \frac{\partial^2 \xi}{\partial x^2}e^{-\frac{i E_p t}{\hbar}} \]  

(30)

By substituting these results in (29) we finally obtain, as expected, the Schrödinger spatial equation (28). Allowing the correlation between relativistic Hamiltonian, in the new conception, and that of quantum mechanics, the following associated operators take place:

\[ \textbf{r} \Rightarrow \textbf{r} \]  

(Position)

\[ \textbf{p} \Rightarrow -j \hbar \nabla \]  

(LinearMomentum)

\[ \textbf{r} \times \textbf{p} \Rightarrow j \hbar \textbf{r} \times \nabla \]  

(Angular Momentum)

\[ \frac{2}{m} \frac{\textbf{p}^2}{m} - c^2 \left( m - M_0 \right) \Rightarrow 2 \frac{\hbar^2}{m} \nabla^2 = c^2 \left( m - M_0 \right) \]  

(Kinetic Energy)

\[ \frac{2}{m} \frac{\textbf{p}^2}{m} - c^2 \left( m - 2M_0 \right) \Rightarrow 2 \frac{\hbar^2}{m} \nabla^2 = c^2 \left( m - 2M_0 \right) \]  

(Total Energy)

As it can be observed from these analogies, only the known quantum operators of energy are slightly modified.

\[ \textbf{V. CONCLUSION} \]

Although the quantitative gain in precise measurements of the energy are not so significant, because Einstein’s energy expressions are a good approach in the most of practical cases to experimental values, in our opinion the theoretical corrections informed in this work for the definitions of mass and Energy given by Einstein are very relevant, from a conceptual point of view in theoretical physics. May be, further measurements of matter characteristics, others than Energy, could lead to find
relevant differences favoring the new dynamical definitions reliant on new mass and energy definitions given in this and in the previous work. On the other hand, in this work we could introduce the relativistic concepts into the Schrödinger Equation in a simple way, thanks the direct dependence on velocity of our energy expressions. We believe that this work has set a simple and direct bridge between Relativistic Theory and Quantum Mechanics, signifying a contribution to the unification of physics, conceptually speaking.

REFERENCES


